WHAT IS BEHIND THE COMPLEX DOMAIN OF PLATO'S QUADRIVIUM?

For my friends, Michelle Rasmussen and Fred Haight

By Pierre Beaudry, 11/11/25

INTRODUCTION

Can you discover the principle of composition of something by looking ***behind*** it? For example: What is ***behind*** the complex domain of Plato's ***quadrivium***? The problem I am attempting to solve is not an easy one, because it comes from a long period of misunderstanding of the complex domain between the Gauss-Riemann-LaRouche approach on one side, and the academic fallacy concocted by Euler and Lagrange on the opposite side.[1]

Even though the problem appears to be as simple as watching the days go by, it is the ancient Greek ***quadrivium*** which is hidden ***behind*** the complex domain of a discovery of principle. However, beware; you may not find what is ***behind*** if you only look above or below. The ***triply-connected motion*** that the ***quadrivium*** involves is as complex as the triple motion of the Earth going around the Sun, daily, monthly, and yearly.

Curiously, my question has to do with the music and geometry of numbers. For example: how can one connect all of the residues of powers of a prime number

[1] See Lyndon LaRouche, ***Visualizing the Complex Domain***, EIR, May 30, 2003. As LaRouche noted: "Plato, arguing from the standpoint of pre-Euclidean notions of physical geometry, defined the concept of 'power,' as reflecting those discoveries by means of which the human mind is able to increase the power of man's willful action upon the universe (e.g., Theaetetus)." (p. 19.)

when they are divided by another prime number? What is the ordering connectivity of the residues of **5 (mod.) 17**, for example, when the increasing powers of **5** are continuously being divided within a three dimensional torus of **17** poloidal waves?

What happens to the so called self-evident fact that prime numbers can only be divided by themselves and by **1**? This complex domain problem of division between prime numbers can only be understood from the vantage point of Plato's notion of what LaRouche identified as a higher "***efficient human power."***

Such a Platonic "***efficient human power"*** can be expressed by the fact that both the Musical Lydians of classical artistic composition and the Arithmetical Gaussian doctrine of biquadratic residues are based on the same three-dimensional torus geometry as the triple motion of the Earth going around the Sun. So, my question is, how can such a combination of Lydians and Biquadratics help us understand the human power of changing the universe? Don't look for mathematical equations to solve that problem; mathematics cannot deal with it.

I believe that the answer to this question can only be found in what Lyndon LaRouche identified twenty five years ago as ***"the means by which mankind has been able to increase…man's power, measurable power, in and over the universe…"***[2] It is worth reading the entire LaRouche statement on this one:

LYNDON LAROUCHE: HOW 'THE UNIVERSE OBEYS LAWFUL COMMANDS'

"Well, typical of those kinds of acts that we make — which we can prove, the universe will obey; otherwise the universe won't obey them — are actions which conform to the discovery of a universal physical principle. If you can discover a validated, universal physical principle, and you can give that, as an order to the universe, the universe will obey. Man is the only creature that can do that! That can formulate an order, called a universal physical principle, validate that discovery, and issue that discovery as ***an order, a command***, to the universe, and the universe is compelled to obey.

[2] Lyndon LaRouche, ***Storm Over Asia, Take Two: I Told You So, and Now It Is Happening***, EIR, September 15, 2000, p. 35.

"That is the means, the accumulation of these principles, which are part of our technological culture, is the means by which mankind has been able to increase the life-expectancy, to improve the demographic characteristics of populations, and, in general, to increase man's power, ***measurable power, in and over the universe, per capita and per square kilometer.*** That's the great, scientific experiment.

"We are able to do this, not only through physical experiments, through physical discovery: We're able to do this, by discovering higher levels of methods of social cooperation, through which, we're able to cooperate in fostering these kinds of discoveries, and applying them.

"So, those things. Those are the kinds of actions, which the universe acknowledges to be ***man's willful actions of significance.*** Everything else that man does, is on the level that any lower form of animal life can accomplish.

"So, therefore, the kinds of action which distinguish a human being from lower forms of animal life, is that, and only that.

"Now, look at this question of strategy, which I've introduced here, from that standpoint: Strategy should mean, once we've understood these lessons — which, presumably, we had learned from study of European history, since the time of Solon and Plato. Say, what's important? What is strategy?

"The purpose of strategy is to defend the human species; to improve its condition, to improve its well-being; to improve its power in and over the universe at large. That's the purpose of strategy.

"In order to do that, we must promote scientific discovery, and utilize it. We must promote those discoveries of principle, such as artistic principles, which enable us to cooperate, in more advanced ways, to utilize these physical discoveries, for man's benefit. What we, therefore, require, is forms of society, in which we perpetuate the rearing of our children, and our institutions, in such a way, that this mission of mankind, implicit in our nature, is fulfilled.

"Thus, we fight to defend this idea of progress. We fight to defend and improve forms of society, which promote progress. We fight to undermine, and nullify, those forms of culture, and political and social systems, which are the enemies of progress. The significance of the United States is that it was produced as a product of a certain phase in European civilization, coinciding with the 15th-Century Renaissance, centered in Italy. It struggled to create a form of society, in which the only legitimate authority awarded to government, was the responsibility and power, to promote the general welfare ***of each and all persons***. That is, to promote progress, in that sense.

"In this process, during that century, the policy was adopted, of having self-governing, modern, sovereign nation-states, whose authority to rule, was located in the commitment to progress so defined. Against that, we had an opponent. The opponent was forces of bestiality: Those, who see a few people, as the power to ***use as human cattle, the majority of other people, other nations, and subject populations, generally***. This is called, oligarchy. So, the forces of progress, and the nation-state are pitted against the forces of oligarchy.

"In the same way that the idea of free trade, of globalization, today: ***These are the enemy.***

"Because, without the nation-state, without protection of the form which only the nation-state can provide for an economy, to ***ensure*** progress, can we have progress. Those who propose to ***liquidate*** the nation-state, that is, to globalize it (or globularize it); those who propose free trade, rather than fair prices to protect the process of production of food, and other things upon which life depends: ***These are the enemies*** of civilization."[3]

[3] Lyndon LaRouche, ***Storm Over Asia, Take Two: I Told You So, and Now It Is Happening,*** EIR, September 15, 2000, p. 35.

THE MUSICAL LYDIAN QUADRIVIUM

As Lyndon LaRouche demonstrated extensively throughout his Music Manual,[4] the most rigorous way to make changes in the musical system, as in the universe as a whole, is with the axiomatic division of the six human voice register shifts, which divide the octaves into two halves. If you divide this first half in half again, you will obtain what I am calling Lydian divisions or a series of four minor thirds.

The location of such double divisions can be found in the first movement of Beethoven's Piano Sonata Opus 27, No. 2, "***Sonata quasi una fantasia."*** (See Figure 3.) Play those few measures on a keyboard and you will hear in your mind how the interaction of the ***half of the half dissonance*** of those two Lydian spiral actions generates the key signature of that composition.

I submit to you the following four Figures 1, 2, 3, 4, which I am introducing as the ***Lydian quadrivium*** for Beethoven's ***Ninth Symphony***. For me, such a ***Lydian quadrivium*** leads to the freeing of the prisoners chained in Plato's Cave, as conveyed by Socrates in Plato's ***The Republic, Book VII***.

Those four Figures raise a lot of questions that I cannot yet answer completely, some of which I submit to the reader for consideration. My questions to you are about the application of Plato's ***quadrivium*** applied to Beethoven's ***Ninth Symphony***. If you have any insight, hypothesis, or opinion, please send me a message at: **pierrebeaudry@larouchepub.com**

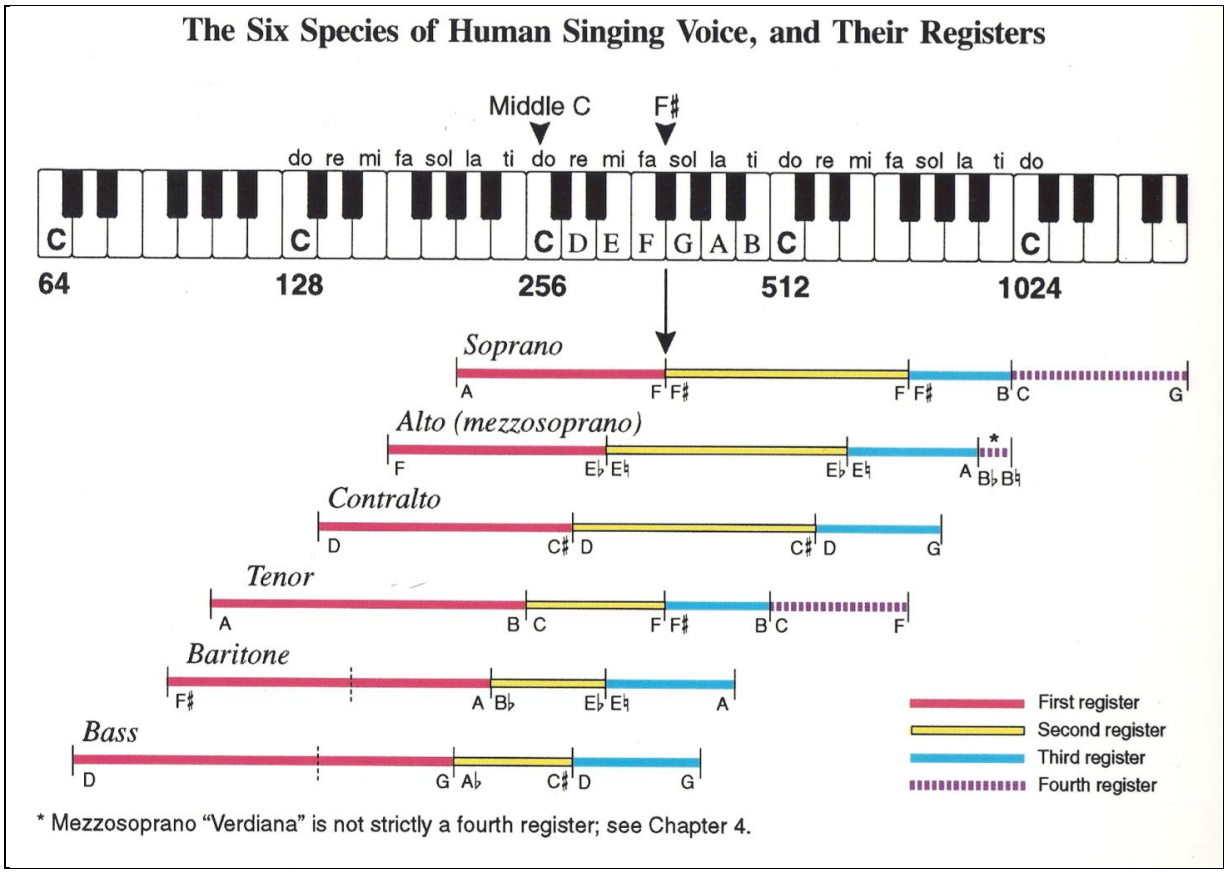
1. Do you see ***behind*** the following four Figures of the Lydian ***quadrivium*** a creative process which can help break the prisoners' chains in Plato's Cave?
2. Do you see ***behind*** Plato's ***quadrivium*** an epistemological setting for putting your life at risk by attempting to save the prisoners from their axiomatic illusions?

[4] ***A MANUAL ON THE RUDIMENTS OF TUNING AND REGISTRATION***, Schiller Institute, 1992.

3. Do you see ***behind*** my Lydian ***quadrivium*** any preestablished harmony such as the ordering of reciprocals inside of torus geometry?

Note that the three last voices of Lyndon LaRouche's illustration of the six species of the human singing voice, from his Music Manual, correspond to the second register of the three male voices (Tenor, Baritone, and Bass) and their geometrical arrangement are the same as the three sets of Lydian spirals of the Lydian ***quadrivium***, such that the geometry of Lydian notes correspond to the geometry of numbers.

THE SIX SPECIES OF HUMAN SINGING VOICE, AND THEIR REGISTERS



TENOR ……. C – F#
BARITONE ….. Bb – E
BASS ……. Ab – D

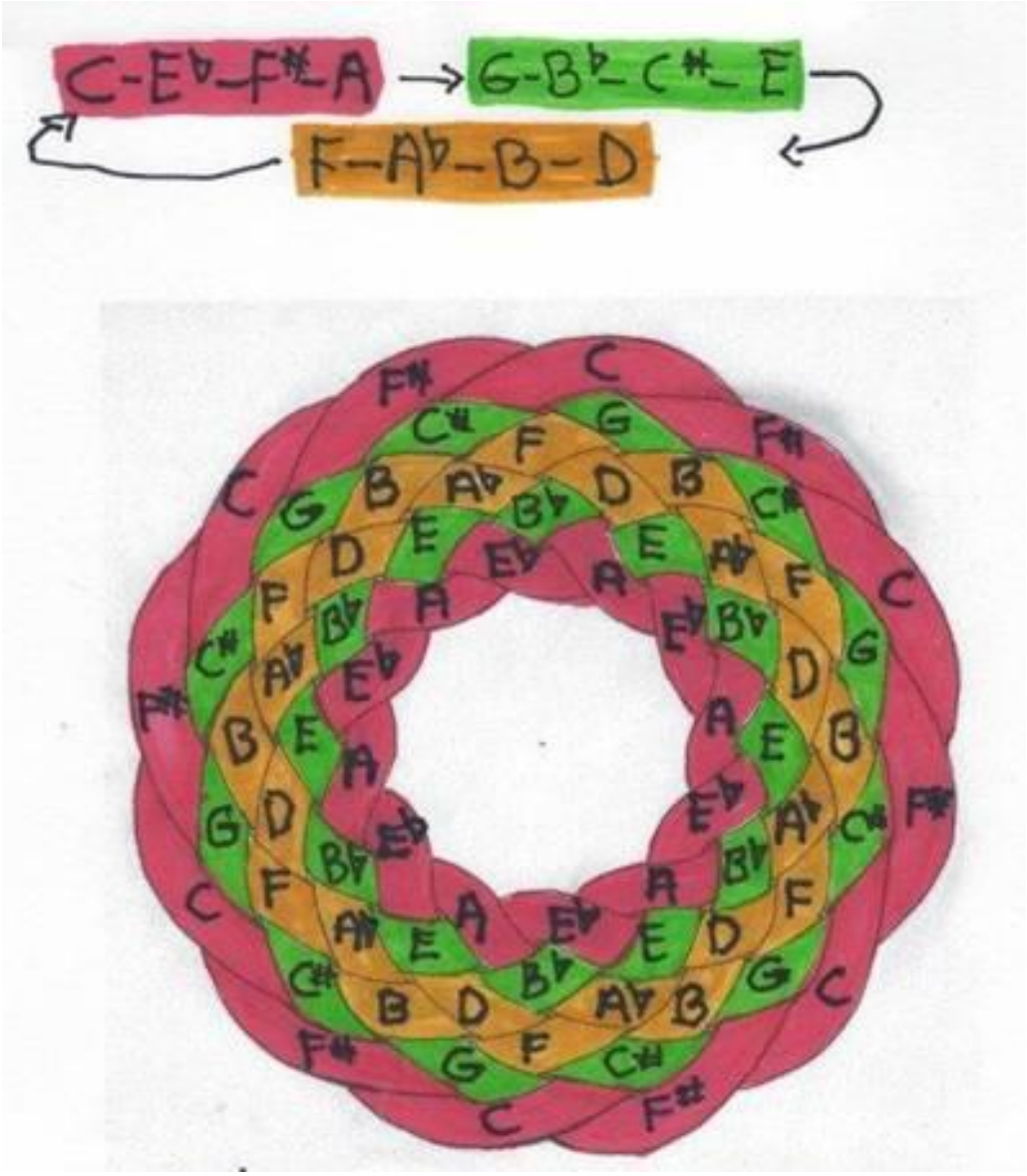


FIGURE 1. GEOMETRY OF THE LYDIANS

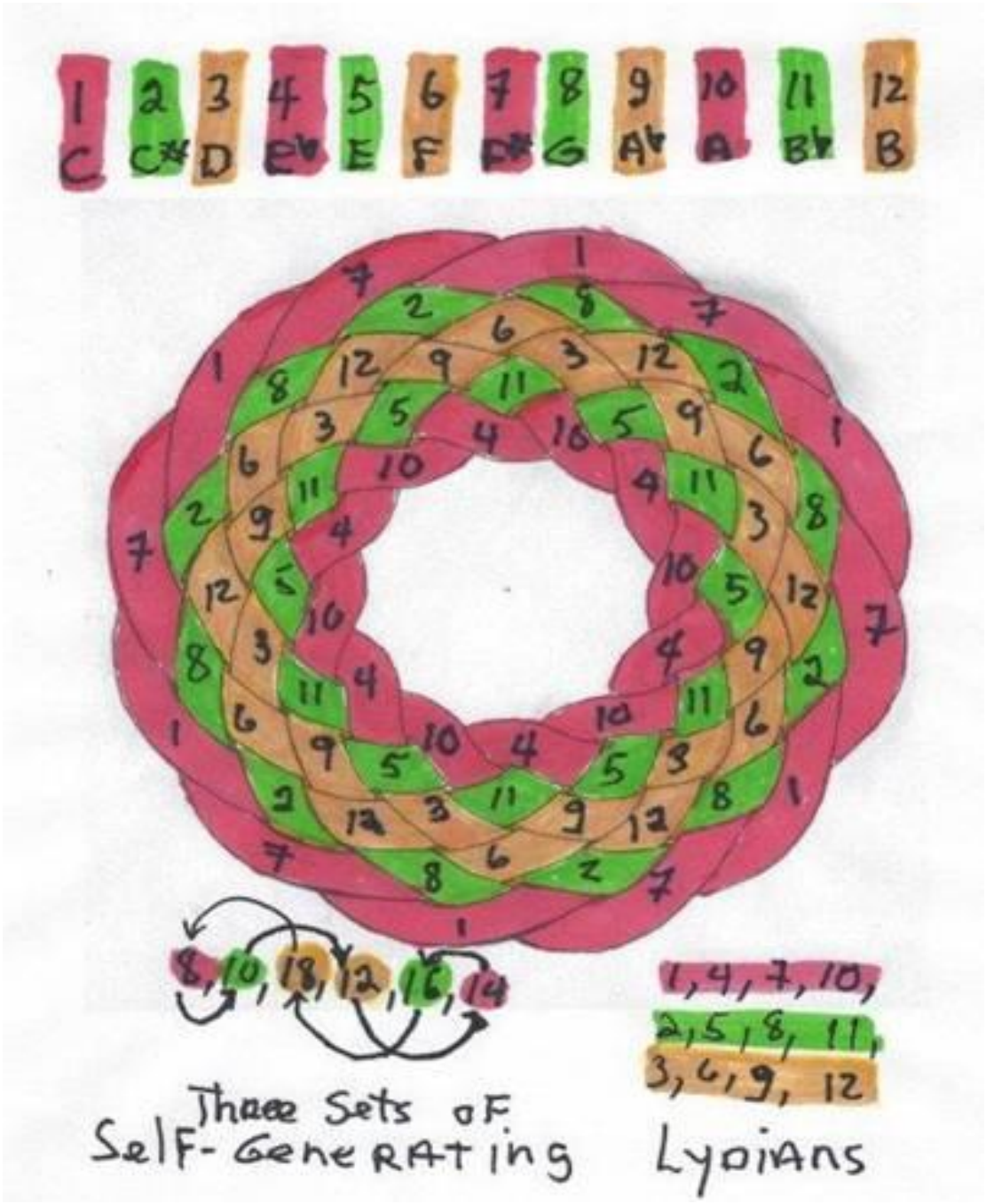


FIGURE 2. ARITHMETIC OF THE LYDIANS

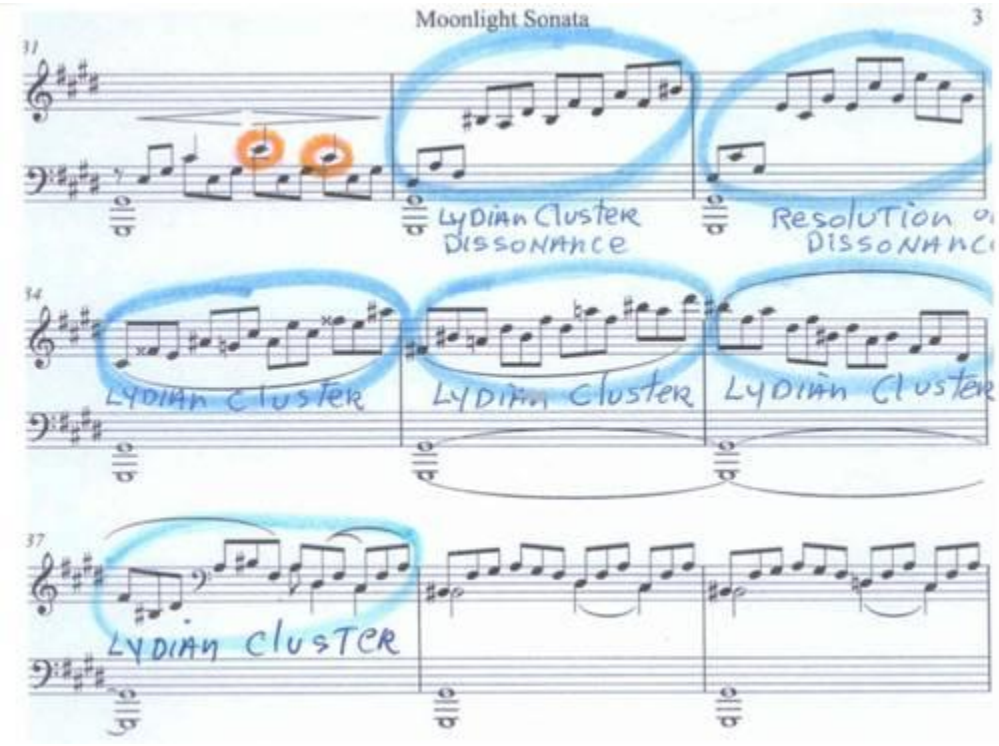


FIGURE 3. MUSIC OF THE LYDIANS

THE PLANETARY ORBITS AND THE EQUAL-TEMPERED MUSICAL SYSTEM
by WILLIAM BOHDAN

PLANETS	ASTRO. UNITS	Log. 10X	ADDED CONSTANT	MULTIPLE CONSTANT	CYCLE EQUIVALENT	MUSICAL CYCLES	PLANETS
MERCURY	(P) 0.310	0.5086	+2.496	x 128.8	255.97	C = 256	MERCURY
MERCURY	(A) 0.470	0.3279	" "	" "	279.25	C#=271.22	MERCURY
VENUS	(P) 0.715	0.1457	" "	" "	302.72	D = 287.35	VENUS
VENUS	(A) 0.725	0.1397	" "	" "	303.49	Eb =304.44	VENUS
EARTH	(P) 0.983	0.0074	" "	" "	320.52		EARTH
EARTH	(A) 1.017	0.0073	" "	" "	322.42	E = 322.54	EARTH
MARS	(P) 1.379	0.1396	" "	" "	339.46	F = 341.72	MARS
MARS	(A) 1.661	0.2204	" "	" "	349.86		MARS
ASTEROIDS	(P) 2.2	0.3424	" "	" "	363.32	F#=362.04	ASTEROIDS
ASTEROIDS	(A)3.6	0.5563	" "	" "	393.13	G = 383.57	ASTEROIDS
JUPITER	(P) 4.95	0.6946	" "	" "	410.95	Ab=406.37	JUPITER
JUPITER	(A) 5.45	0.7364	" "	" "	416.33		JUPITER
SATURN	(P) 9.006	0.9545	" "	" "	444.43	A = 430.54	SATURN
SATURN	(A)10.074	1.0032	" "	" "	450.69	Bb=456.14	SATURN
URANUS	(P) 18.288	1.2622	" "	" "	484.05	B = 483.26	URANUS
URANUS	(A) 20.092	1.3030	" "	" "	489.31		URANUS
NEPTUNE	(P) 29.799	1.4742	" "	" "	511.36		NEPTUNE
NEPTUNE	(A)30.341	1.4820	" "	" "	512.37	C = 512	NEPTUNE

FIGURE 4. ASTRONOMY OF THE LYDIANS

THE HIDDEN DIFFICULTY BEHIND BIQUADRATIC RESIDUES

Unless an axiomatic problem becomes resolved in time during a process of development of the human species, the growing progress of that development of mankind must come to an end. This may not be the best way to end things, but is there a way to prevent it from happening? Although I cannot explain why this biquadratic residue problem can prevent this from happening, but, I know that the way to do it is by going through the process of discovering why it is not impossible to figure out, when you look ***behind*** it.

Let me start with the most difficult part, which is to demonstrate that the process of resolving Gaussian biquadratic residues is similar to explaining, astronomically, the dual Poloidal-Toroidal motion of the Earth orbiting inside of the Solar System.

You may not be able to divide prime numbers by each other directly, but you can do it indirectly through the ordering of the residues of the divisions of their powers through a third higher dimensionality. In other words, you can only divide primes by each other through the residue cycle of powers of what Plato identified as his ***stereonomic quadrivium***.[5]

The biquadratic residue process of arithmetical calculation involves two different arithmetical and geometrical motions, which involve the continuous dividing of one prime number by another prime number. The secret of solving such a curious problem is located in the arithmetic and geometric function of dividing the powers of one prime by another and taking their residues, that is their differences, as the common source of their shared unity of agreement.

Let's say that the number of Earth's daily rotations per month is 5 Poloidal rotations and the number of monthly orbits around the Sun is 17 Toroidal rotations per year. How many continuous motions can those two Poloidal and Toroidal prime numbers resolve between each other inside of a cyclical stereonomic torus? It cannot be done without accounting for the differences, or residues, of all of the increasing powers of 5 when divided by 17.

[5] See Plato, ***The Republic, Book VII***.

Let me explain this curious complex operation with the following arithmetic calculation table of residues taken from **5 (mod.) 17**. The divisions are not visible:

1,
1x5 = 5,
5x5 = [25 – 17] = 8,
5x5x5 = [125 – (7x17)] = 6,
5x5x5x5 = [625 – (36x17)] = 13,
5x5x5x5x5 = [3125 – (183 x 17)] = 14,
5x5x5x5x5x5 = [15625 – (919 x 17)] = 2,
5x5x5x5x5x5x5 = [78125 – (4595 x 17)] = 10,
5x5x5x5x5x5x5x5 = [390625 – (22977 x 17)] = 16,
5x5x5x5x5x5x5x5x5 = [1953125 – (114889 x 17)] = 12,
5x5x5x5x5x5x5x5x5x5 = [9765625 – (574448 x 17)] = 9,
5x5x5x5x5x5x5x5x5x5x5 = [48828125 – (2872242 x 17)] = 11,
5x5x5x5x5x5x5x5x5x5x5x5 = [244140625 – (14361213 x 17)] = 4,
5x5x5x5x5x5x5x5x5x5x5x5x5 = [1220703125 – (71806066 x 17)] = 3,
5x5x5x5x5x5x5x5x5x5x5x5x5x5 = [6103515625 – (359030330 x 17)] = 15, etc.

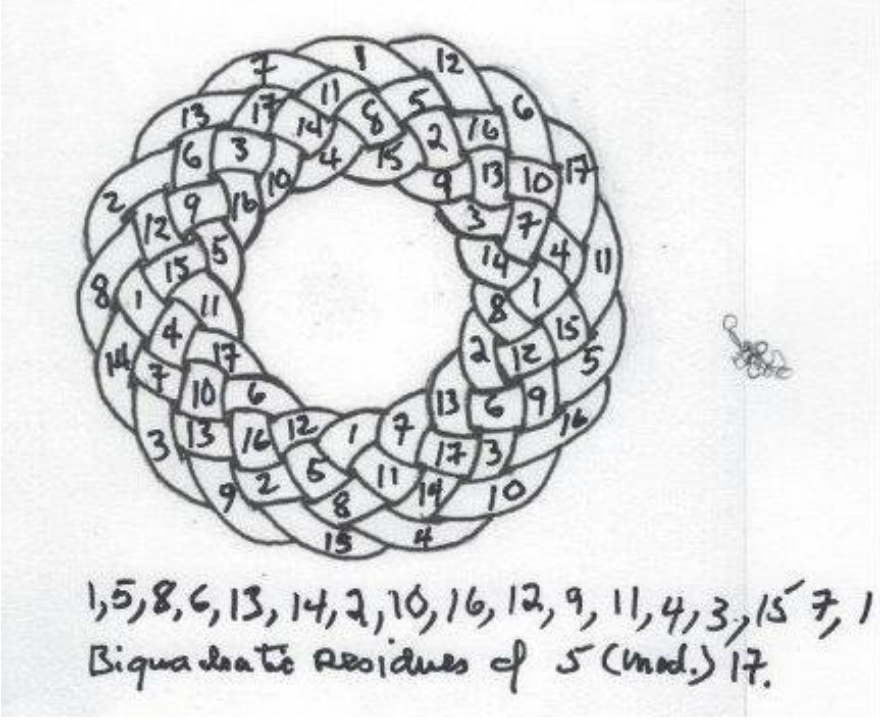


Figure 5. Biquadratic residues of 5 (mod.) 17.

The solution to this complex domain problem can be found simply by filling in all of the empty holes of the following ***cyclical stereonomic torus*** (Figure 5.) with **4** sets of the same successively repeated series of **17** numbers.

The beauty of this ***arithmetic-geometric solution*** lies in its simplicity; that is, in the fact that you can foretell the coming next future residue number simply by counting your steps and without making any calculations whatsoever.

The geometrical identification of all the residues of the powers of **5 (mod.) 17** can easily be obtained simply by locating the residues on the outer rim of the torus. All you need to do is to follow the wave motion of the ribbon going around the torus from outer rim to outer rim, starting from **1**, and move clockwise to **5** around the small circumference of the torus **(1x5 = 5)**. Next, five poloidal wave motions later **(5x5 = [25 – 17] = 8)** will take you to residue **8**.

This means that one complete poloidal wave rotation around the small circumference of the torus represents one additional power of **5** around the toroidal orbit of **17**. Every other such completed rotation corresponds to an additional power of **5** as identified in the calculation table above.

In other words, if you wish to find the first three residues of **5, 8,** and **6,** for example, your finger must go around the small circumference of the torus once, starting at **1** in order to get to **5**; then **5** more waves in order to get to **8**, then again**, 8** more waves in order to get to **6**. And, so on.

You can even find an easier way to complete these cycles simply by counting the number of rings on the outer rim of the torus until you reach the number which corresponds to the number of your counting process. For example, starting from **6**, you can get you to the next residue **13** simply by counting **13** rings along the outer rim, then count **14** rings in order to get to residue **14**, and then count **2** rings to get to **2**, and so on. It is that simple.

Who would have thought that such a complex Gaussian domain of biquadratic residues was geometrically so easy to construct? My question again is: how can this planetary orbit process be expressed in music?

A MOST CURIOUS ASPECT OF THE MUSICAL SERIES OF C-256

Consider that in the second half of his ***Ode to Joy,*** Beethoven asks the reader a crucial question that he must attempt to answer while listening to his composition which is: "***How should you run your race?"***

All creatures drink of joy
At nature's breasts.
All the Just, all the Evil
Follow her trail of roses.

Kisses she gave us and grapevines,
A friend, proven in death.
Ecstasy was given to the worm
And the cherub stands before God.

Gladly, as His suns fly
through the heavens' grand plan
Go on, brothers, your way,
Joyful, like a hero to victory.

Be embraced, Millions!
This kiss to all the world!
Brothers, behind the starry canopy
There must dwell a loving Father.

Are you collapsing, millions?
Do you sense the creator, world?
Seek him above the starry canopy!
Behind the stars must He dwell.

How do you find the Lydian creative process ***behind*** such quadratic stanzas of the ***Ode to Joy***?

When the listener starts from these final stanzas, and begins to investigate what dwells ***behind*** the stars, he must ask himself: how can I discover "***the loving Father***" who is "***behind the starry canopy.***" However, how can this be done when you are chained in Plato's Cave chasing shadows?

This is when the reader must make an axiom busting decision to break with his own chains and actually start investigating what the final causality of something like Beethoven's ***Ninth Symphony*** is all about. He must start from what is beyond his sense of hearing and go ***behind*** the shadow dominated aspect of his mind to look for something that he cannot see and hear with his eyes and ears, but only with his mind. He must break with his previously deceptive mental axioms and discover the principle of a more sophisticated geometry, which will change his way of thinking entirely. That is how you can change the past from the future so that your present becomes different from what it used to be.

After those chains are broken, the joy of his discovery of principle that made the prisoner break away, which is the real subject of Beethoven's composition, is located in the preestablished harmony of the underlying unity of the composition, which can only be discovered and captured by the mind from ***behind*** the scene.

As Dennis Small recently identified in Velasquez's ***Las Meninas*** painting, the real subject of the work is not what you see with your eyes in the foreground, but what your mind sees in the background ***behind*** the visible scene, which is the ordering of the creative power of discovery itself. If what you see in the background is a portrait of the King and Queen in the mirror, you have missed the point. You must also look ***behind*** that background scene.

The same applies to the real subject matter of Beethoven's ***Ninth Symphony***. In other words, the real subject the listener has to look for is not in what he sees and hears, but in what his mind discovers as the principle of unity of the composition ***behind*** the background**.** That is not obvious at all, and yet, that is what we have to look for and have to discover from the end of Beethoven's entire composition; that is, from the words and the musical score of the ***Ode to Joy***.

LYNDON LAROUCHE: 'VISUALIZING THE COMPLEX DOMAIN'

Consider the following ***quadrivium*** hypothesis: each note of a musical composition is 1) rotating on itself, 2) orbiting around the small poloidal circumference of a torus, and 3) orbiting around the large toroidal circumference of that same torus, because the **C-256** series can be visualized as a cyclical musical

system. I found this to be a truthful hypothesis because the musical series of **C-256** is composed of **6** octaves of **12** notes each covering **6** closed cycles. Therefore, the **C-256** series is not a linear domain, but ***a complex cyclical domain*** like the Solar System. This observation came to me from the simple examination of the following series:

1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, etc.

1, 2, 4, 8, 7, 5, 10, 11, 13, 8, 7, 14, 19, 20, 22, 26, etc.

1, 2, 4, 8, 7, 5, 1, 2, 4, 8, 7, 5, 1, 2, 4, 8, etc.

If you look at this growing process in a linear fashion, you will not find the answer to the question of what is ***behind*** the **C-256** series. However, if you project all of the numbers of that series into a ***cyclical stereonomic torus***, you will discover that the process of alternatively going under and over has a completely closed and preestablished harmonic ordering in which all of the numbers located at the extreme ends of all of the inner and outer diameters are ***reciprocals.*** That, to me, represents the hidden unity of this process of change in music: ***reciprocity throughout all of the cycles of the C-256 series.***

The reason the **C-256** series has cycles of **6** units of action is because the three dimensional domain has six different directions of motion: upward and downward, forward and backward, clockwise and counterclockwise. You can demonstrate that by adding up the series of whole numbers into a geometrical torus in such a manner that you add an empty space after each additional number.

Insert the series of numbers: **1+02+003+0004+00005+000006+0000007**, etc. until you have filled in all of the empty spaces of Figure 6. The same principle of alternate motion of going successively under and over, applies to all of the octaves of the **C-256** series. For example, take the following case of the **P/T = 3/16** torus.

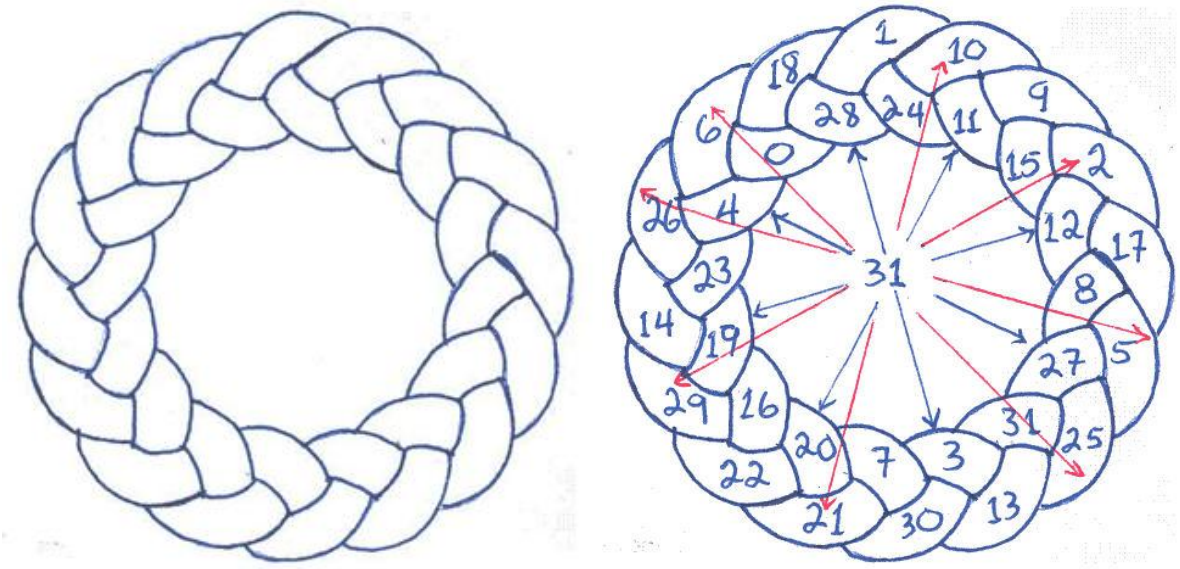


Figure 6. Reciprocity of the **C-256** series. Each of the **16** diameters is a reciprocal of **31**.

The step by step ordering of this ***cyclical stereonomic torus*** is simply based on adding an additional empty unit of action after each new step taken. This raises the question: What is the ordering principle of action underlying this simple process of adding an increasing number of empty spaces after each new step? The answer is: the unity of all of the extreme ends of the diameters of Figure 6, is reciprocity; that is, the ***One Behind the Many*** as, for instance, the resolution to Nicholas of Cusa's ***Coincidence of Opposites***.

The reason for creating all of those empty space units after each new step is because those empty spaces must be made available for future numbers. Thus, the anticipation of such a future is in the temporality of the ***time-reversal simultaneity of temporal eternity.***

The beauty of this ***cyclical stereonomic torus*** is that it permits you to foretell how many numbers are coming from the future and which one will be located where as the motion goes forward. In other words, you can know in advance which Lydian number is going to go where, because the only arithmetical series, which permits you to have such knowledge of the future is the musical series of **C-256.**

Such is the hidden domain of the ***One Behind the Many***, which can be found in such a ***cyclical stereonomic torus ordering process*** of the musical octave geometry of numbers.

A similar non-visible ordering process resides "***Behind the stars where He must dwell."*** This will remain forever unknown unless you discover that numbers are geometrically reciprocal, musical, and astronomical, simultaneously, as is the power of the human mind to make new and more encompassing discoveries of principle. Beethoven's answer to the question of what must dwell ***behind*** the stars comes in the stanza before last, in the ***Ode to Joy***, which states:

Even the worm can feel contentment,
And the cherub stands before God!
Gladly, like the heavenly bodies
Which He sent on their courses
Through the splendor of the firmament;
Thus, brothers, you should run your race,
Like a hero going to victory!

How can one express that musically, otherwise than by looking for what lies ***behind*** the creation of the universe? That is the beauty ***behind*** Plato's complex domain of the ***quadrivium.***

GOD'S IDEA BEHIND THE CREATION OF THE UNIVERSE IS THE CREATION OF MAN

Ask yourself a simple question: Why did God Create the Universe? What did He have in mind? Did He want to play ball with solar systems and galaxies and see how far He could get away with cycling them around? I don't think so. What God had in mind was the creation of Man; that is, a creature who could think like Him. That is the reason why God made Man in his own Image because He wanted to make him discover how he could master His method of composition, which I can only poorly describe as: **SPACE-TIME-REVERSAL IN THE SIMULTANEITY OF TEMPORAL ETERNITY.** That is what is ***behind*** Beethoven's Ninth Symphony.

Michelle Rasmussen wrote a beautiful report on this subject, ten years ago, titled, ***'All Men Become Brothers': The Decades-Long Struggle for Beethoven's***

Ninth Symphony, which I want to introduce, here, with great joy, because of the tremendous insights she had about Beethoven and Schiller. Michelle wrote:

> ***"Beethoven, drawing on Friedrich Schiller, the great poet of freedom, gave us a gift, not only for German speakers, but for all humanity. And, as pointed out by Lyndon LaRouche, to compose such a gift, it is not the love of music, but the love of humanity, which is the source of the passion the musician, no, the human being, draws from. Beethoven, himself, wrote that his call was 'to use my art as a means of relieving needy humanity.'***
>
> ***"Yes it is out of this passion, pouring out in tones, that the human moves his fellow men and women to look inside of themselves, to find their own fount of creativity, compassion, and yearning to make an immortal contribution to all of humanity.***
>
> ***"Let us begin our story at the end, and let us end at the beginning.***
>
> ***"Listening in retrospect, you can partake in the future yet to be.***
> ***'…God, like one of our own architects, approached the task of constructing the universe with order and pattern, and laid out the individual parts accordingly, as if it were not art which imitated Nature, but God, himself, (who) had looked to the mode of building of Man who was to be.' [Johannes Kepler, Mysterium Cosmographicum.]"*** [6]

This statements is a perfect example of time-reversal thinking because it does not proceed from a simple sense perception expectation, but from a complex form of events governed by ***time-reversal simultaneity of temporal eternity***; that is, from the past, present, and the future ***behind*** the scene, in the background of creative intention, which is located in all places at once. In other words, what you hear is not what you hear; it is what you hear and it is also what is underlying the composition. As Michelle put it:

[6] Michelle Rasmussen, ***'All Men Become Brothers': The Decades-Long Struggle for Beethoven's Ninth Symphony***, EIR, June 26, 2015, p. 38.

"***The (Ninth) symphony is a good example of musical time-reversal in a single masterpiece—playing with time, backwards and forwards—where the unfolding of the music, as it is being performed, is driven by the concept of the future, completed unity, which is present, at all times, in the minds of the conductor, such as Furtwängler, the musicians, and the listeners.***

"Furtwängler self-consciously named this phenomenon. He described it as the tension between the 'Nahören' (near-hearing), the music that is being heard right now, and the 'Fernhören' (far-hearing), the future, unified, completed musical idea. The two intersect at every moment, and the tension between listening to the present from the future, and the future from the present, is what gives the composition, as it is being performed, its gripping, driving quality, creating a 'dynamic quality of musical space-time.'

"Beethoven was also self-conscious of this when composing. '[M]y custom when I am composing even instrumental music is always to keep the whole before my eyes,' and work towards realizing it. He once made a note to himself to sketch out all of the voices as they appeared in his mind, not only the theme, to get accustomed, at once, to the concept of the whole."[7]

LaRouche would probably have called this process the ***One of the Many in the simultaneity of temporal eternity***. In fact, Michelle quotes LaRouche as saying the equivalent of this in the following space-time measuring process:

"***Instead of imagining space-time as measuring displacements according to a fixed reference-scale, imagine that forward displacements of a space-time-matter continuum change the ostensibly linear scales employed for the preceding moment. Imagine that this change in the basis-measurement***

[7] Rasmussen, ***Op. Cit,*** p. 38.

> ***is of such a form that instead of measuring displacement of the developing continuum according to linear (scalar) magnitudes of increments of time, distance, mass-energy, and so forth, that the scale of measurement is a series of numbers determined by an exponential function. That is perhaps the best heuristic representation of the general idea for today's ordinary informed consciousness. It also expresses precisely the consequence of Beethoven's approach to composition in the late quartets and related works.***"[8]

But, Michelle goes a step further with LaRouche's idea of coupling the dual motion of ***Freude/Freiheit*** relationship like the Poloidal-Toroidal motion of a torus. It is the joy of discovery which causes freedom, and the freedom to discover which causes the joy. This is also what is ***behind*** the complex domain of Plato's ***quadrivium:*** the passing from the instrumental (orchestra) to the human voice (chorus) in Beethoven's ***Ode to Joy*** creates a higher dimensionality that Michelle identified in the following way:

> "***After another dissonant disturbance, harkening back to the very beginning of the movement, Beethoven makes his revolutionary non-linear upward jump—the instrumental recitative from before, is now superseded by the startling entrance of a human voice, in the middle of a symphony! In the transcendental passage mentioned above, the lone baritone, in the same tonal range as the cellos and basses, starting with the beginning notes from victory march emerges, sung by a soldier and his comrades, leading into a battling double fugal orchestral variation based on two variations of the theme, which foreshadows the choral double-fugue to come. This is followed by the chorus at full throttle***." [Rasmussen, p. 44.]

[8] LaRouche, ***The "Florestan Principle" in Art***, New Solidarity, January 7, 1977.

This axiom busting change is fundamental because it is as if there were a dimensionality missing during the first three movements of the ***Ninth***; and then, suddenly, the human voice is introduced, like the creation of Man by God inside of an already existing physical universe. That is the concept of JOY that Schiller was talking about when he wrote:

> ***"There is something mysterious in the effect of music, that it moves our inner self, so that it becomes a means of connection between two worlds. We feel ourselves enlarged, uplifted, rapt—what is that called other than in the domain of Nature, drawn to God? Music is a higher, finer language than words. In the moments, where every utterance of the uplifted soul seems too weak, where it despairs of conceiving more elegant words, there the musical art begins. From the outset, all song has this basis."***[9]

BEETHOVEN'S ODE TO JOY IS THE SOLUTION TO PLATO'S QUADRIVIUM

Plato's ***quadrivium*** is not simply putting together geometry, arithmetic, music and astronomy; it is not four separate domains of knowledge lined up next to each another; it is, from a more advanced standpoint, the science of artistic composition, that is, the doctrine which uses art and science together as single ***One Behind the Many*** in order to deal with the false shadows of knowledge that we are born with and have to struggle with during all of our lives.

The purpose of the ***quadrivium*** is to bring joy and happiness to mankind. It is the scientific art of breaking our prisoner's chains in Plato's Cave by making people discover for themselves how the different parts of the elephant in the room are composed and how they can be put together.

[9] Michelle Rasmussen, **'All Men Become Brothers': The Decades-Long Struggle for Beethoven's Ninth Symphony ('Alle mennesker bliver brødre':Den årtierlange kamp for Beethovens niende Symfoni) - Schiller Instituttet**, EIR, June 26, 2015, p. 51.

However, this is a very scary and dangerous pedagogical method to adopt because you must risk your life in order to accomplish it. People will bite your head off if you push too much. So, how can this life and death dilemma be resolved? The best person to answer that question is Socrates who identified the danger of his axiom busting method as follows:

> "***Perhaps when I examine your statements I may judge one or another of them to be an unreal phantom. If I then take the abortion from you and cast it away, do not be savage with me like a woman robbed of her irst child. People have often felt like that toward me and been positively ready to bite my head off for taking away some foolish notion they have conceived. They do not see that I am doing them a favor. They have not learnt that no divinity is ever ill-disposed toward man, nor is such action on my part due to unkindness; it is only that I am not permitted to acquiesce in falsehood and suppress the truth***."[10]

Why is that statement so devastatingly true? First of all, this is only true for someone who is aware that he is applying the method of axiom busting. It is a mystery to everybody else. Secondly, the reason why this method is devastating is because the victim in that situation is not the prisoner whose axioms are being busted, but Socrates. And Socrates is upset, because he thinks he may have done something wrong by trying to help someone else. Who gave him the right to judge someone else's statement? That is the axiom busting issue of the ***quadrivium***. Who has the right to tell anyone else what is right and what is wrong?

In point of fact, ultimately, no one has the right to break the prisoners' chains and to force them out of the cave, because this is something the prisoners have to do for themselves. This is a free choice that each prisoner has the power to accept or to reject, because the principle of reason can only be a personal discovery and cannot be imposed from the outside by someone else. The greatest joy,

[10] Plato, Plato, ***Theaetetus, 151 c-d.,*** translated by F. M. Cornford.

therefore, comes when someone makes this discovery by himself and for himself, because that is the discovery of the principle of human self-government.

This means that the discovery of principle of Plato's ***quadrivium*** is not only a personal matter, but it cannot be discovered unless the prisoner comes to maturity and takes the personal responsibility to break his own chains. That is what the matter of life and death is all about ***behind*** Plato's ***quadrivium***, and that is why its most profound significance is located ***behind*** Beethoven's ***Ode to Joy*** in the ***Ninth Symphony.*** However, this raises more questions than answers, some of which both Michelle Rasmussen and Fred Haight are better able to answer than I am.

Fred identified the underlying characteristic of Beethoven's double-fugue as being the function of the Chorus and the content of two stanzas whose opposition causes the "***joyful spark of divinity.***" That opposition can be found within the first half of the six quadratic stanzas which anticipates the joy of the spark of divinity:

Freude, schöner Götterfunken,
Tochter aus Elysium,
Wir betreten feuertrunken,
Himmlische, dein Heiligtum.

Deine Zauber binden wieder,
Was die Mode streng geteilt,
Alle Menschen werden Brüder,
Wo dein sanfter Flügel weilt.

Chor: Seid umschlungen Millionen!
Diesen Kuß der ganzen Welt!
Brüder – überm Sternenzelt
Muß ein lieber Vater wohnen.

Joy, beautiful spark of divinity,
Daughter of Elysium,

We enter, drunk with fire,
Heavenly One, your sanctuary.

Your magic binds again,
What fashion has strictly divided,
All people become brothers,
Where your gentle wing rests.

Chorus: Be embraced, you millions!
This kiss to the whole world!
Brothers –behind the starry canopy
A loving Father must dwell.

I see and hear these stanzas as double-doubles, that is, as quadratics similar to what are found in Lydian dissonance resolutions in Cusa's ***Coincidence of Opposites***. As Fred Haight stated about the double-fugue:

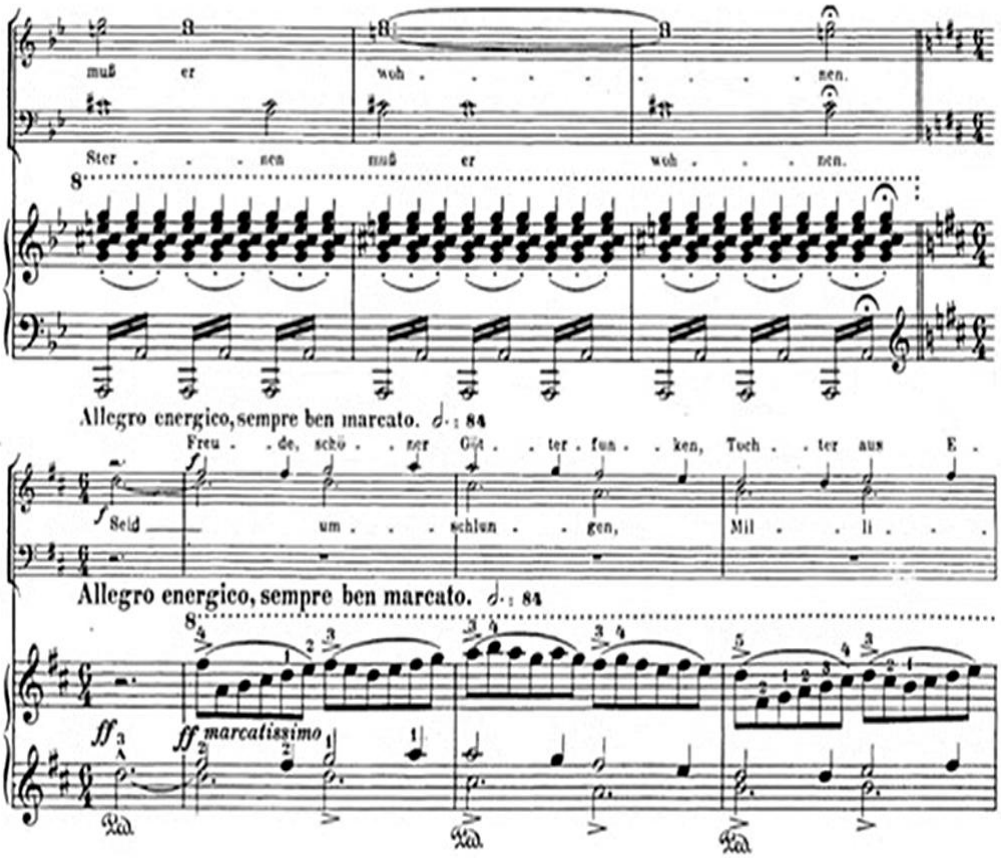
> "In the great double-fugal works by Beethoven, including the ***Finale*** of Opus 106 (***Hammerklavier***), the ***Finale*** of the ***Ninth***, the ***Grosse Fugue***, and the ***Finale*** of the ***Credo*** of the ***Missa Solemnis***; there are two subjects that go in and out of formal double-fugues, but the movement always consists of a dialogue between those two subjects. In some cases, such as the ***Finale*** to the ***Credo*** of the ***Missa Solemnis***, which we discussed, and hopefully listened to last time, those two ideas work together for the good.
>
> "In the ***Grosse Fugue***, they clash bitterly. As in a true marriage, or relationships between hostile states, both sides must change themselves and each other, in order to survive. The ***Grosse Fugue***, if played well, is a sublime example of such change. 'Beethoven recognized a potential in the double-fugue far beyond anything his predecessors did. It could not be contained within a formal double-fugue. It had to embrace all of human experience.'" [***A Discussion of Beethoven's Ninth Symphony*.**]

This last statement by Fred has tremendous implications for the present world strategic crisis. Since "all of human experience" is reflected in Plato's ***quadrivium***, similarly a double-fugue, which is a double-double, has to have the

same quadratic characteristic function of dissonance resolution as the three sets of Lydian spirals in Beethoven's ***Ninth***.

This same principle also applies to the difference between the American System and the Oligarchical System; it is the hidden underlying principle ***behind*** the British opposition to the government of the people, by the people, and for the people: ***Are you innocent until you are proven guilty, or are you guilty until you are proven innocent?*** This is the life and death question behind the current world crisis. Is this also the issue behind the conditional release from prison of Former French President, Nicholas Sarkosy?

The language of the ***Ode to Joy*** has three quadratic stanzas which also reflect the clash and resolution of that self-governing principle. The double-doubles in the musical score, as Fred noted in the following piano score: "where the voices at the top sing **A, C#, E, G,** and the piano's right hand treble clef plays **B, Bb, C#, E, G**, with the base note of the left hand being a low **A**."



Fred answered that question in the following manner about this double-fugue dissonant resolution in **beetschill3.mp4 - Google Drive.** Note especially the section that he identified as **"*The Lydian dissonance. The great tension, in anticipation of a breakthrough.*"** [16:00 / 1:38:30]. As he reported, the Lydian dissonance is located between two subjects:

"Fugue subject 1:
Freude Schoner gotterfunken Tochter aus Eklysium,
Wir betreten feuertrunken Himmlische dein Heiligthum

Second Fugue Subject:
Seid umschlungen Millionen deisen kuss der ganzen welt."

"A double-fugue has two fugue subjects of equal weight that development plays against each other. The first fugue subject of Beethoven was derived from the first eight lines of the first stanza, and the Ode to Joy theme is derived from that and is associated with that. […]

"The second subject is a little trickier. Beethoven has taken the words from the first two lines of the third quadratic stanza. […] Schiller is attempting to determine the relationship between the action of the Chorus and the action on the stage. This is what Pierre Beaudry would call ***dividing by half and by half again.*** It's total anticipation of the breakthrough which lies ahead.[…]

"If that's complex and difficult: good! It shows that humanity has achieved much more than most people think, and it's, therefore, even the more worth defending.

"If our political leaders understood only one percent of what is going on, here, instead of worshiping Ozzy Osbourn, they might not be so willing to put the world at risk. And grasping the full magnitude of it can give us the inner strength to fight, and keep fighting and not give it up. If humanity can produce this kind of beauty then, as Lyndon LaRouche said, "***It's the law***." [22:44]

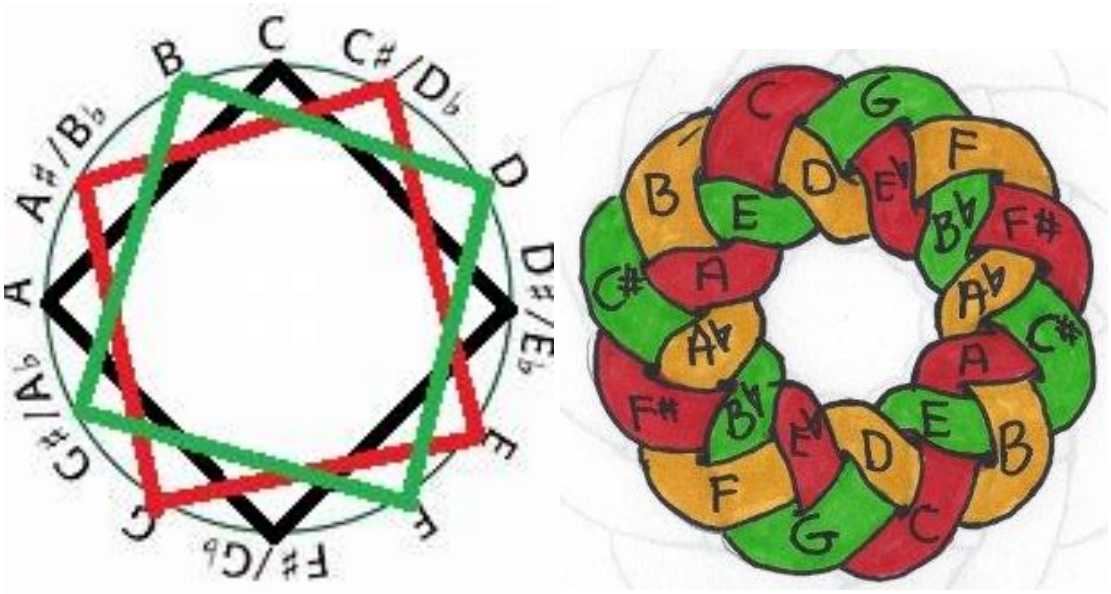


Figure 7. Fred's "Chromatic circle spirals," which I have transposed into three Lydian Spirals: **[C, Eb, F#, A], [G, Bb, C#, E], [F, Ab, B, D].** [20:30]

Furthermore, I might add that such great joy of mankind can become generalized when international world leaders discover that such a ***quadrivium*** principle is recognized as the true self-governing principle of a unified humanity at peace with itself. That is the reason why **Dag Hammarskjöld**, former Secretary General of the UN, always used to play the 4th movement of Beethoven's ***Ninth Symphony***, every year, on UN Day, from, October 24, 1953 until his death on September 18, 1961.

THE RIEMANNIAN MANIFOLD GENERATING THE LYDIAN MUSICAL INTERVALS

When you introduce music to Riemann's manifold from the vantage point of Plato's ***quadrivium***, you discover that the Lydian configuration of a forth and three minor thirds, that is, of seven notes **G, B, D, F, D, B, D…**, which, when played on a keyboard in that succession, have a dissonant arrangement calling to be resolved in the immediate future with the addition of a new unknown note which you can

only hear in your mind. This is like foretelling the immediate future from a higher One from ***behind*** the scene.

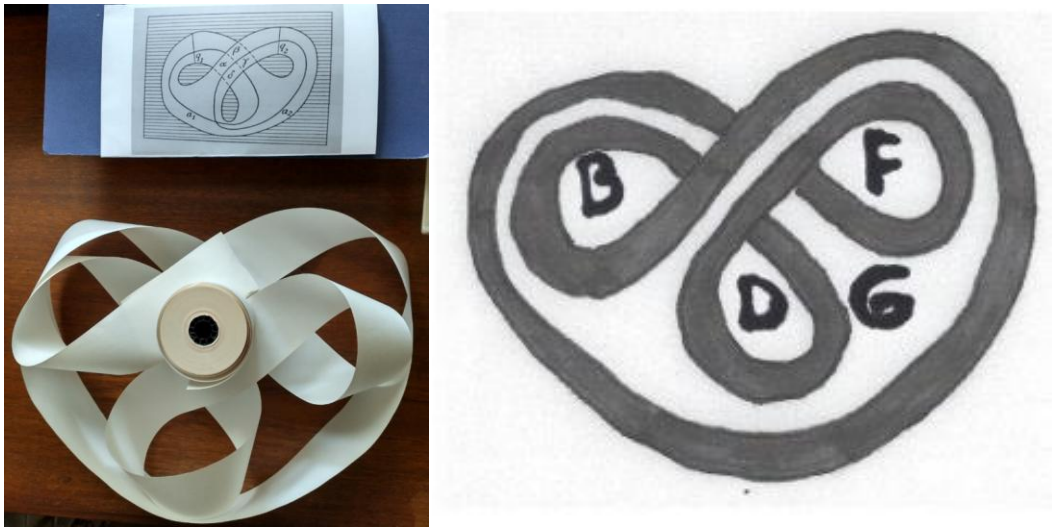
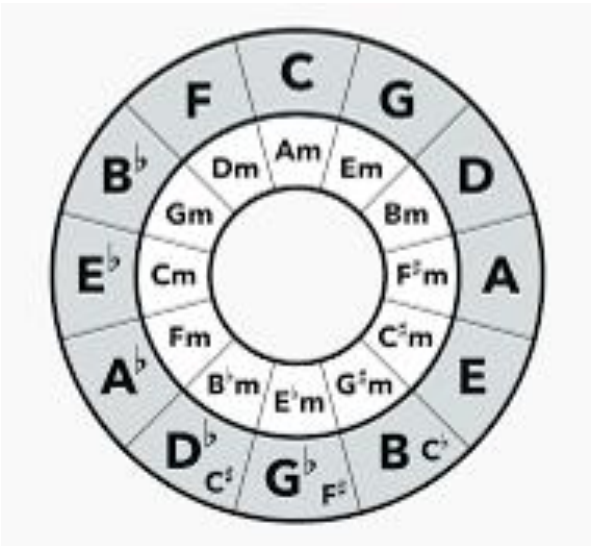


Figure 8. Riemann's dissonant Manifold generating the Lydian musical foretelling of the future: **G, B, D, F, D, B, D** generating **C**



That new note is **C,** which then forms the completely resolved biquadratic series of **G, B, D, F, D, B, D … C.** That biquadratic is then followed by a second biquadratic resolution **C, E, G, Bb, G, E, G … F**, which, itself, calls for the next one to come from the future, which is **F, A, C, Eb, C, A, C … Bb**, and so forth. Thus, you are generating a completely closed series of the musical octave, which is the inversion of the circle of fifths: **C, F, Bb, Eb, Ab, Db-C#, Gb-F #, B-Cb, E, A, D, G,** in which each fifth comes from the future by time-reversal.

Figure 9. The Circle of Fifths and the Performative Lydian Cycle.

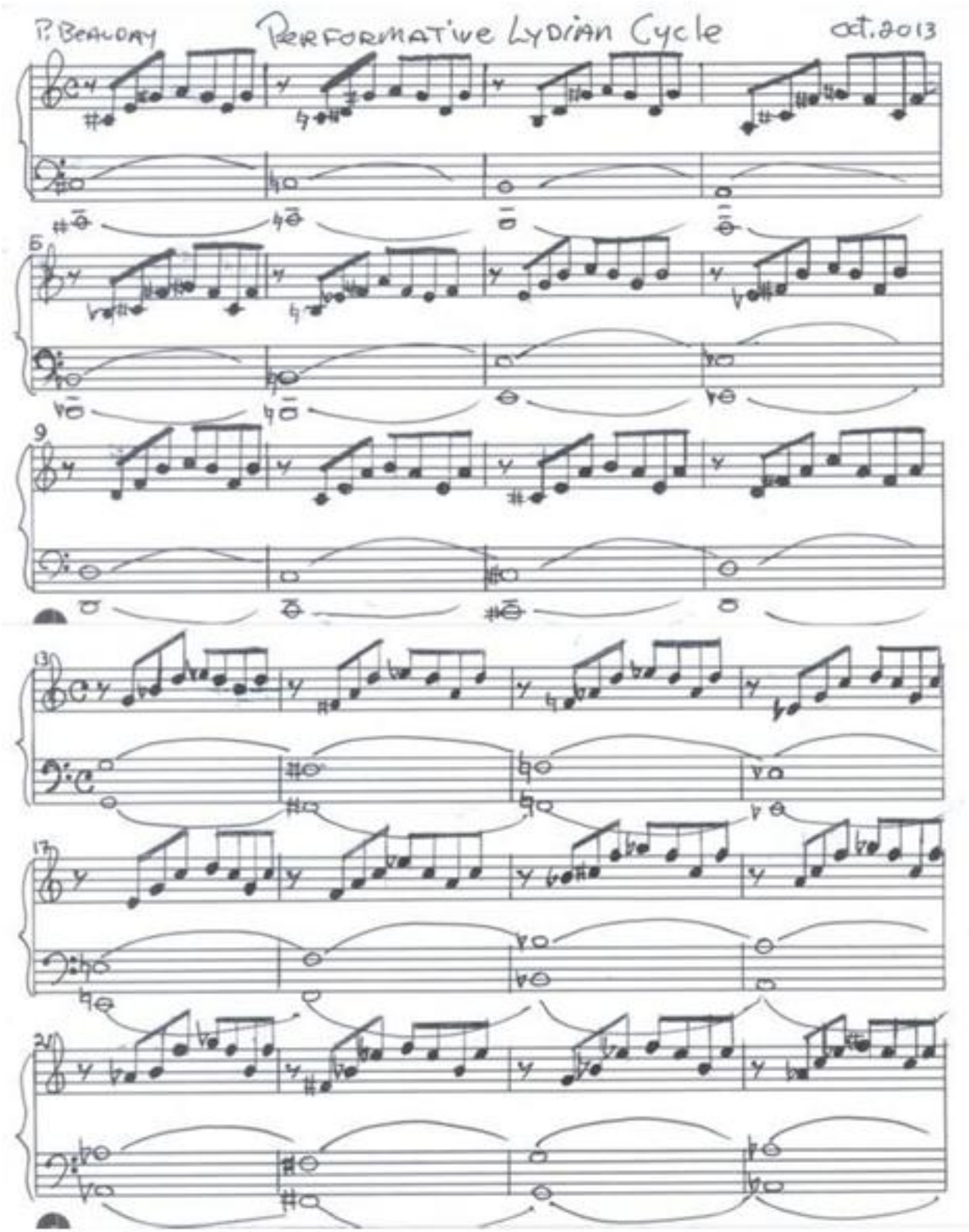


Figure 10. Performative Lydian Cycle.

The Riemannian dissonant manifold generates Lydian quadratics as follows:

B, Eb, F#, A, F#, Eb, F# generates **E**
D, F#, A, C, A, F#, A generates **G**
F, A, C, Eb, C, A, C generates **Bb**
Ab, C, Eb, F#, Eb, C, Eb generates **C#**

Consequently, the three Lydian spirals generate one another in the following manner:

[E- G- Bb-C#] generates in succession the keys of **D, F, Ab,** and **B**
[D- F- Ab - B] generates in succession the keys of **C, Eb, F#,** and **A**
[C- Eb- F#- A] generates in succession the keys of **E, G, Bb,** and **C#.**

Thus, such a Riemannian Lydian series not only anticipates the future, but it also generates the complete musical system as the ***One Behind the Many*** through the circle of fifths. So, finally, we can conclude this report by asserting that it is the generating principle of such Lydian series which is ***behind*** Plato's ***quadrivium***. This is what LaRouche used to call having fun.

THE GEOMETRY OF NICOLAS OF CUSA'S LEARNED IGNORANCE

Plato wrote at the entrance of his Academy: **"Let no one enter here unless he tries to master geometry."** This is the reason why Socrates was reminding us that in order to solve the mystery of Plato's Cave, we had to do the impossible. As he stated: "***It would be hard to find a science which demands more efforts to learn and to practice than the one of measuring numbers.***" [11]

You may be convinced that it is impossible to do four things at once, but try to master this one. Socrates just stated above that there is a way to do the impossible, which is to be found in the method of "***measuring numbers***." However, if you wish to ***measure numbers*** in the following Figure 11, you must

[11] Plato, ***The Republic, Book VII, 526c***.

construct their invisible geometry yourself, because their true relationships are otherwise not discoverable without reconstructing them, yourself.

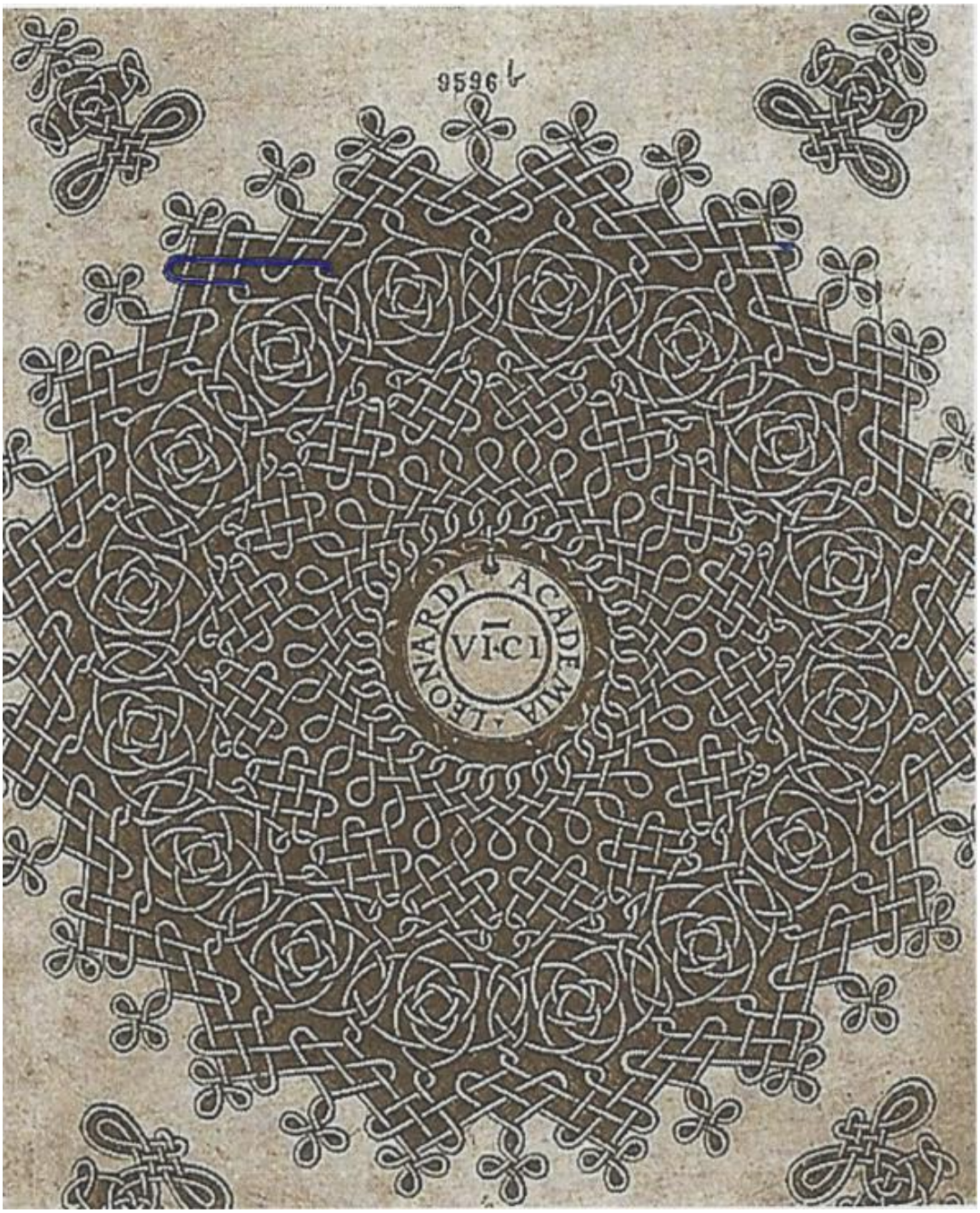


Figure 11. The visible geometrical Emblem of Leonardo Da Vinci's Academy shows four different interconnected motives.

I'll give you a hint: the numbers you are looking for are reflected as the ***One Behind the Many,*** underlying the four sets of overlapping circular motives of Leonardo Da Vinci's Academy Emblem. But, beware; some of the shadows of what you see in Figure 11 are actually numbered and shaped differently than the geometry you are looking at in Figure 12.

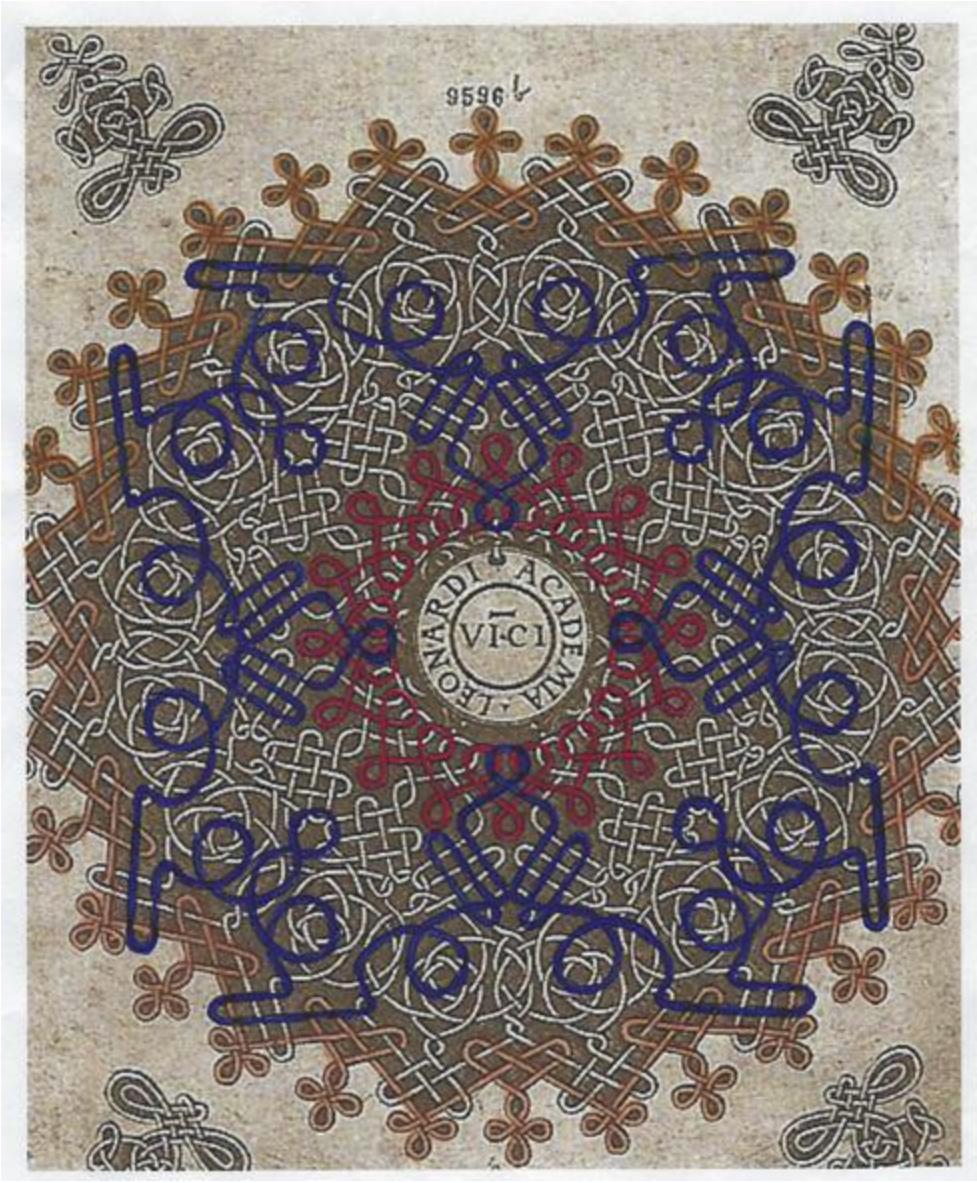


Figure 12. The invisible geometry of musical loops behind Leonardo Da Vinci's Academy Emblem shows the unifying principle behind three different and separate voices unified as a single ***One Behind the Many***.

Why does Figure 11 have **4** motives and Figure 12 has **3** voices? The Leonardo Emblem was obviously created for the investigative mind, because a mere first visual glance at it is deliberately deceptive. The Emblem ***is changing within itself.*** The astute investigator will discover that the Emblem is based on the **C-256** musical series revealing only three overlapping voices producing four different motives. This is the Lydian process of Cusa's ***Learned Ignorance***.

The entire Emblem is geometrically divided ***by half and half of the half***. The first inner red voice has a motif of **32** closed loops and a second outer yellow voice has a motif of **128** closed loops. Between them, a third single closed set of a blue voice of **8** sitting figures contains **64** circular loops and **4** sets of such a motif generate a total of **256** similar blue loops. The whole process, therefore, represents the resolution of the ***One Behind the Many*** of a series of octaves of the musical system representing a coincidence between seeing and hearing.

Who would have thought that the ***One Behind the Many*** of Leonardo' Emblem was a combination of geometrical cycles of musical octaves [**4, 8, 64, 256 (blue), 16, 32 (red), 32, 128 (yellow)**] of the musical scale? By composing such an interlacing and overlapping Emblem, Leonardo confirmed what Plato was saying in ***Republic, Book VII***, and demonstrated that he had, himself, discovered the Lydian geometry of the **C-256** series as a geometrical form of measuring numbers in the way that Socrates suggested.

Next, Figure 13 below shows a more simplified version of the same Platonic principle of the three dimensional geometry of numbers that Leonardo displayed as the musical basis for his own ***quadrivium*** of Figures 11 and 12. This is the geometry of the ***Advantage of the Other;*** that is to say, the geometry of Nicholas of Cusa's ***Learned Ignorance*** whereby the art of leaving more and more room for others to develop in the future is based on the principle of reciprocity of ***One Mankind***.

Since Figure 13 represents the process which best exemplifies, geometrically, the resolution of Cusa's ***Coincidence of Opposites*** by means of fusing together all of its reciprocals, I propose that you construct that reciprocal cycle for yourself, as en exercise, by inserting the series of counting numbers into

Figure 13 below, following the process of jumping a new empty space after every number. Start counting anywhere you wish and you will get the desired result.

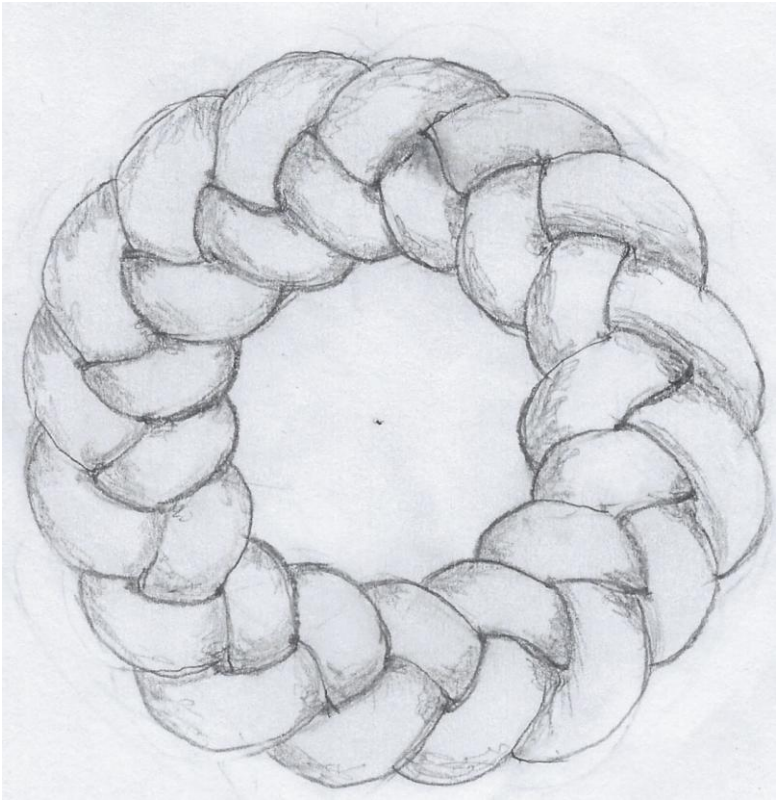


Figure 13. Poloidal/Toroidal torus of P/T = 3/16.

We are all dependent on the geometry of numbers; and if we treat them respectfully and geometrically with reciprocity, we'll be more than happy to ***leave behind the same opportunities that were left open to us for the sake of others coming in the future***. Pope Leo XIV, on October 30, 2025, showed a crowd of 10,000 pilgrims in St. Peter's Square at the Vatican, how that can and should be done with the method of ***Learned Ignorance*** of Nicholas of Cusa.[12]

However, the truth of what one has to say is going to become demonstrable only when what one sees coincides with what one hears; that's the hidden proof of one's own personal discovery of principle behind Plato's ***quadrivium***.

[12] ***Pope Leo: Cusa's 'Coincidence of Opposites' Gives Hope for Peace***, EIR, Vol. 52, No. 44, November 7, 2025.

CONCLUSION

Finally, there is one last interesting paradox in Plato's ***quadrivium,*** which appears in both music and in statesmanship more generally; but, which cannot be found anywhere in politics or in wheeling and dealing. You can solve it by using the fundamental method of Nicholas of Cusa's ***Coincidence of Opposites***, as I have just indicated above. All you have to do is to divide all opposites as the inversion of the **C-256** series; that is, divide ***by half and half of the half***.

This means that in order to go from a lower to a higher voice register, you have to find the higher ordering geometrical principle ***behind*** the scene of the differences between two opposing voice registers. Understanding the differences between two such dissonant voices is the same thing as understanding what divides people coming from different countries. This geometry gives you a means to resolve all of the oppositions by going to a higher level of understanding. This doesn't require more diplomacy, but more truthful statesmanship. This is the problem facing the American people, today, with respect to the Russian people and the Chinese people.

In a sense, an American and a Russian are like two prime numbers; they cannot be divided by each other without leaving some remainder behind them. However, what if there were to be something common between their differences? What if someone were to be able to organize those residues in such a way that reciprocity could be found between them? If such reciprocity can be discovered in the torus geometry of biquadratic residues, then, why couldn't one discover a similar ordering principle among the differences between civilized people? Could this happen in any other way but by dividing the poloidal powers of one by the toroidal cycle of the other?

Moreover, would that be able to secure peace on Earth? Perhaps not, but that could bring hope for mankind, when at least three out of the six voices are governed by a common ability to resolve such Lydian dissonances and differences among one another, and become resolved to show how this works to other people.

FIN