
GILLES PERSONNE DE ROBERVAL'S DISCOVERY OF AXIOMATICS BEHIND THE SINE-CYCLOID SURFACE

Why the proof of an axiomatic change does not require mathematics

by Pierre Beaudry, July 20, 2023

FOREWORD

There are little known moments in history when the whole of mankind is affected by an axiomatic transformation such as with the Italian Renaissance, which caused a multitude of human beings to be transformed for the better. We have come to a similar historical moment today.

Such special moments are blessed with extraordinary joy but also marred with tremendous upheaval, because the prevailing public opinion control over society shatters under the breakdown of underlying assumptions that are demonstrated to be dangerous and false. The question today is: will the current demise of the neo-liberal colonial unipolar world come to an end with peace or with war?

Around 1637, Gilles Personne de Roberval made a discovery which can be called the *sine-cycloid surface discovery of principle*, which illustrated how the pathway of such an axiomatic change was able to change the way human beings look at themselves and the universe. Since everything that moves in the universe takes the path of least action, let's examine the following Roberval pathway.

The cycloid is known as a curve traced by a point fixed on the circumference of a circle, which rotates along the surface of a horizontal plane. Such a curve is also generally explained by mathematicians as a way of measuring the Leibnizian infinitesimals that Roberval called "indivisibles." However, Roberval's true discovery is the discovery of principle of the *sine-cycloid surface*, which not only

expresses the concept of the infinite, but also increases the power of the human mind by causing an axiomatic change.

In fact, Roberval's discovery not only has the power to change the way you think about yourself and the universe, but it also gives you the power to change the universe itself.

THE PATHWAY OF DISCOVERING THE AXIOMATICS OF THE SINE-CYCLOID SURFACE

The infinite, here, is not to be expressed by the fallacy of the concept of exhaustion, as Archimedes argued in his approximate integration of the area of the circle with an extremely large polygon. As Cusa later demonstrated, circles and polygons are two incommensurable species and one cannot determine the nature of the other. On the contrary, Roberval's construction of the area under the cycloid curve is a genuine integration of what he called his infinitely small "indivisibles," because it encompasses the totality of the area.

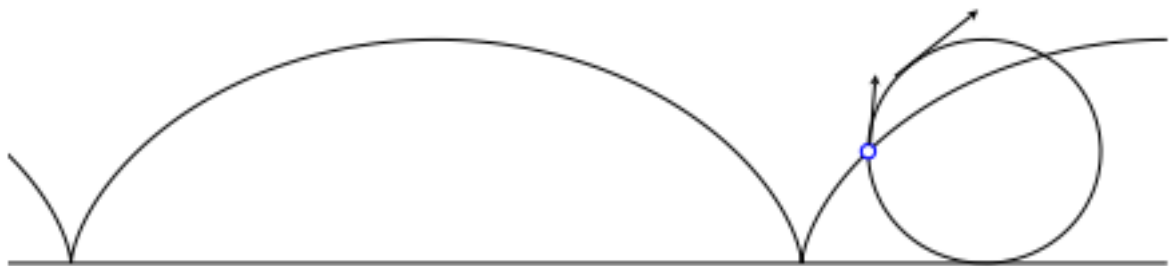


Figure 1. The partial mathematician view of the cycloid curve

Not only is Roberval's discovery of principle a genuine epistemological integration between straight lines and curved lines, it is also an integration between different spheres of knowledge, most notably between geometry, artistic composition, and epistemology, because the discovery includes, simultaneously, four discoveries into a single one: 1) the discovery of the cycloid curve, 2) the

discovery of the sine curve, 3) the discovery of the areas between both of them, and 4) the discovery of the effect that Cusa's isoperimetric closure has on increasing the power of the human mind. It is in that sense that this pedagogical is an axiom buster.

Imagine that the circle is located in a state of the past and that the cycloid curve, sine curve, and the surface between them are all three located in the future, which does not yet exist, and which is about to be discovered. This is like the complex motion of three double musical Lydian dissonances which move toward their resolutions in a Beethoven artistic composition, in the simultaneity of physical eternity. Construct the following experiment for yourself with Beethoven in the back of your mind:

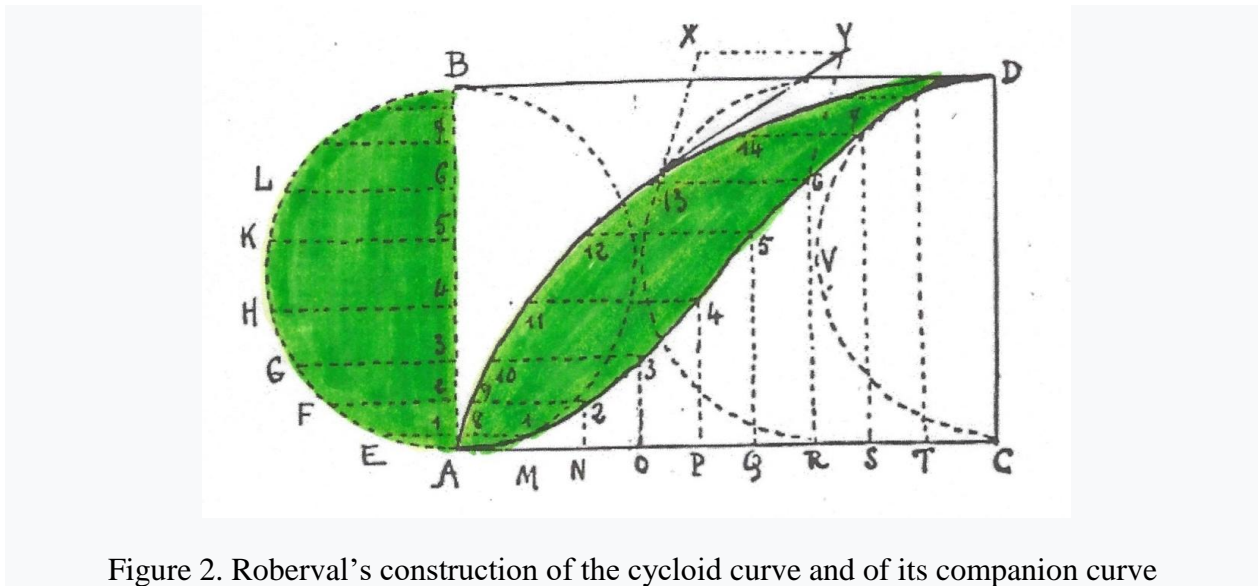


Figure 2. Roberval's construction of the cycloid curve and of its companion curve

Rotate the circle (Figure 2.) clockwise, and move it along line AC from left to right; that is, from the past to the future, by one half of a rotation. What happens to your mind when the circle goes through the transformation of everything you see inside of the new space located within rectangle ABCD? Roberval explained this complex motion of transformation as a new idea of measurement called "indivisibles," as Bonaventura Cavalieri identified them in 1635. Roberval wrote:

"Then I take the same sines E1 , F2 , G3 , etc. and I carry them on each height found on each diameter, and I draw them towards the circle

[from the companion curve], and from the [two] ends of these sines are formed two lines [curves], of which one is A 8 9 10 11 12 13 14 D [the cycloid], and the other A 1 2 3 4 5 6 7 D [the companion curve of the cycloid].”¹

The area of one half of a circle is changed into a *sine-cycloid surface* by a half rotating action of the circle as a result of which, the entire area under the cycloid curve can be determined to be precisely equal to three times the area of the generating circle. The proof does not come from the arithmetical sum of indivisibles, but from the geometrical redistribution of the totality of indivisibles included in the circle. Thus, the axiomatics of measurements have changed.

This discovery is not difficult because it only requires paying attention to the reason why the area under the half-cycloid corresponds to three different halves of the generating circle. This can be found in the following manner and by construction alone, without making any mathematical calculation whatsoever. An axiomatic change does not require any mathematics.

The key to this “mystery” lies in discovering the relationship between the sine curve and the cycloid curve. Once the mind discovers that the area between the two curves is equal to half the area of the generating circle, then one can conceptually and descriptively solve the rest of the puzzle by dividing the whole process by half again, and thereby discover the irony of the paradox of the *sine-cycloid surface*.

Since the sine curve divides the area of the rectangle ABCD into two equal halves (ABD5A and ACD5A), and both of those halves can also be divided into two more equal parts, all four areas ACVD5A; DVCD; A5DVCA; and ABD12A are each equal to the half circle A5BKA. Prove this by construction:

¹ Quoted by Jesse Hamer, *INDIVISIBLES AND THE CYCLOID IN THE EARLY 17TH CENTURY*, MATH 464WI; HISTORY OF MATHEMATICS WITH DR. RICHARD DELAWARE, p, 60. Indivisibles were first discussed as a geometrical method of integration by Bonaventura Cavalieri in, *Geometria indivisibilibus continuorum nova quadam ratione promota* (1635).

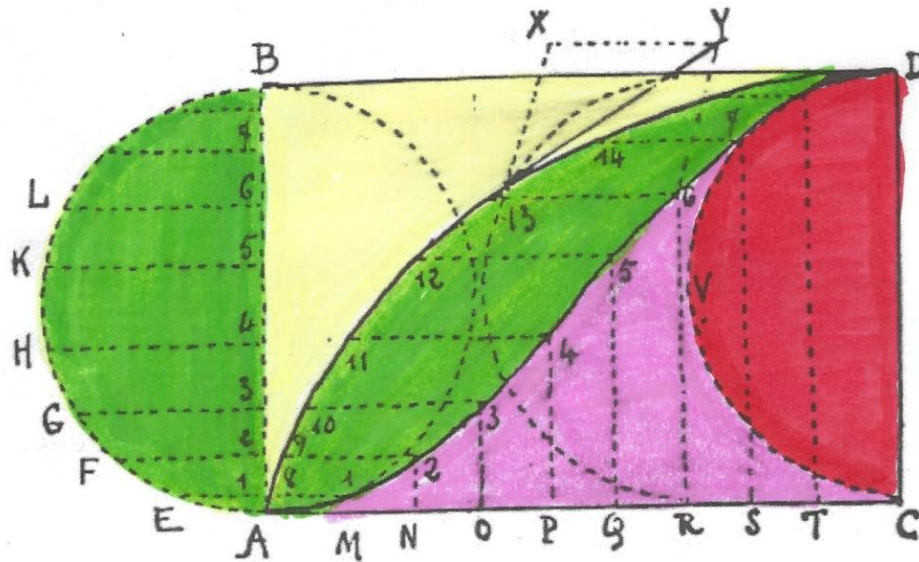


Figure 3. The five areas $ACVD5A = DVCD = A5DVCA = ABD12A = A5BKA$ are all equal to each other because $A5DVCA$ is equal to $A5BKA$.

The three colored surfaces (green, pink, and red) under the half cycloid curve have the same isoperimetric area because each one is bounded by the same dual motion of two different points; one moving up along the diameter and the other moving also upward along half of the circumference of the rotating circle. It is the action which is the key: *It is the geometrical action which brings closure to the sine-cycloid surface of transformation, because the two simultaneous motions of two different points trace the same boundaries as those of the half-circle and of the diameter of the generating circle.*

Following Nicholas of Cusa, such areas represent a coincidence of opposites: the three areas under the cycloid are the same because they are bounded equally by the same isoperimetric action.² In fact, Cusa's isoperimetric principle is very similar to the double musical Lydians generating future resolutions of musical ironies through key changes in a classical artistic composition.

The paradox to be resolved, therefore, is the following: how can the same perimetrical action produce three different areas? You don't have to prove this

² See my report:

[GENERATING THE FIVE PLATONIC SOLIDS WITH NICHOLAS OF CUSAS METHOD OF FOLDING UNFOLDING AND ENFOLDING](#)

mathematically; you simply have to do it yourself, performatively; that is, by constructing it yourself.

WHEN THE DISCOVERY OF THE POWER IS THE POWER OF DISCOVERY

At this point, the question should arise how does one think about the increase of power with the cycloid curve, the sine curve, and the surface between them that are created through a single continuous motion?

Here, the tricky part of this discovery is the inversion which occurs when two different curves are discovered to be generated simultaneously by two points located at two different places on the same circle during a complete circular action: one point (D) is tracing a sine curve (blue) by moving up and down the diameter of the circle, the other point (P) is tracing a cycloid curve (red) by moving along the clockwise or counterclockwise motion of the circumference of the same generating circle (Figure 4) in a way similar to the generation of the area of circle itself. While going through that rotation, the entire area of the rotating circle is completely transferred within the area between the two curves.

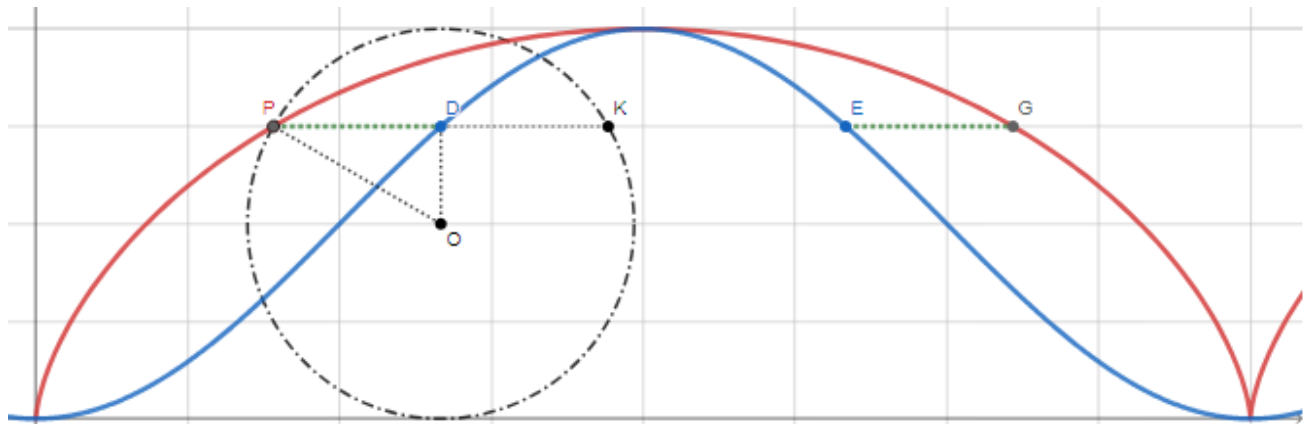


Figure 4. [Cycloid and Its Properties Related Curves - Mathematics Pages \(anirdesh.com\)](http://www.anirdesh.com)

The mental impact of such a discovery is very similar to the joy of discovering the creative process that Gaspard Monge made his students discover at

the Ecole Polytechnique with his principle known as the pedagogical method of the “Monge Brigades.”³

The mental result is that it causes your mind to soar as if it had increased its power to change the universe. The reason and cause of such a power come from the immortal joy of truth; that is, the truth whereby an individual human mind is able to construct a discovery of principle which benefits all of mankind, and which humanity will never forget as long as it exists. Such is the joy of the “pursuit of happiness” in the simultaneity of temporal eternity as expressed in the American Declaration of Independence.

On the other hand, when the circle is not considered as a unit of rotation but as being statically closed thing onto itself, its area can neither be changed within its own self-contained axiomatic domain, nor can it be modified by any sort of fictitious method of exhaustion brought from the outside. It is limited, finite, and closed on itself.

The mind, on the other hand, has the freedom to travel outside of the circle itself in order to move beyond its own limitations. However, it also has the ability to remain a slave to the public opinion consensus of the mathematical rule of law and remain in its previous sterile state of a phony aggregation of a large polygon. The secret of this discovery lies in how to divide the action by half, and by half of the half again.

In that case, the circle enters into a state of axiomatic transfiguration by rotating its surface outside of itself. The rotating contact with other surfaces and other curves causes the power of the circle to change the space-time dimensionality of its working area. That's creativity, which brings circular action into a completely new dimensionality of existence, where *your conception of the past circle is no longer what it used to be; it has changed, axiomatically, from past to future and it has tripled its power by reproducing itself into three different ways in*

³ See Pierre Beaudry, [*The Metaphor of Perspective*](#), Fidelio Magazine, Vol. IV, No. 2, 1995. “What Monge did, as a student of the Oratorians, is to devise a curriculum which was oriented toward replicating the creative discoveries of the past for the purpose of immediate applications in the military field. Monge and Carnot were able to develop the students’ creative powers so rapidly that students would learn in three months what others would take three years to learn. Organized along military lines, these became known as the Monge brigades.”

the simultaneity of temporal eternity under the cycloid curve. Such is the extraordinary power of the axiomatic change of Roberval's *sine-cycloid surface* discovery of principle.

Such a process of axiomatic change is an example of a *paradigm shift*; that is, a moment of change which not only supersedes the previously stale and self-centered state of mind, but also introduces, by means of circular action alone, a higher conception of what the mind can do.

By making that mental shift, the new discoveries of lines and surfaces become similar to, yet incommensurable with, the previous way of looking at the circle and at the universe as a whole. The action of this constructive measuring principle changed everything. As LaRouche once said: "*Believe nothing that for which you cannot give yourself a constructive proof.*"

When a new *paradigm shift* takes place in your mind in this fashion, the previous state of the world is not only superseded, it also becomes inadequate and false, because of its obvious axiomatic limitations. Such is the difference between Euclidean and Riemannian geometry. Euclidean geometry is not bad; it is simply not as universally valid as some people pretend it to be.

Roberval's epistemological experiment became such a formidable mental obstacle for René Descartes, Newton, and their followers, that it was as if a declaration of an axiomatic death sentence had been pronounced against their view of the world. However, the knowledge of such a new discovery of principle may have opposite effects on different people. At the time of its discovery, it had a very sobering and humbling effect on Roberval himself, Pierre de Fermat, and Blaise Pascal. But, the same discovery had a very nasty effect on Descartes and Newton among others.

The isoperimetric principle of Roberval's discovery establishes a perfect closure of the entire area under the cycloid curve without the Archimedean exhaustion fallacy of composition which used to define the circle.

TRANSFIGURATION

What sort of transformation takes place during this single complete rotation of the circle? Strangely enough, while the right half of the circle rotates along line AC, the left half of the circle becomes transformed into a stretched *sine-cycloid surface* as it is being transferred under the left half of the cycloid curve. Meanwhile, the right half of the circle has not changed at all as it moves during this half rotation. Why not? Because only the right half of the circle can change the left side of the circle.⁴ Here, the reader may be a little perplexed.

However, if you pay attention to the unique time factor involved and to the change of the left and right sides which takes place during the action, you will note that an inversion has taken place without your noticing it. *The action of the right half of the circle changes the left half; after which, the left half of the same circle changes the right half in order to complete the cycle of the sine-cycloid action. What happens to your mind when those two halves of this complete circular action take place successively and without any discontinuity?*

While the circle rotates on the right half of itself, it changes something on the opposite left side of itself. The question is: why is the right half of the circle not also changing? The reason is because the circle is waiting until point A has reached point D before giving to its left half the ability of transforming its right side in the same continuous manner as it did from the right. This inversion is actually avoiding the paradox of changing yourself while you are changing something else. Chronological time cannot deal with that. Indeed, how can a circle change half of itself in one direction and change its other half in the opposite direction, at different times? Why are you required to go through two different moments of space-time in the same action?

It is only after point B has rotated down to point C, and after point A has rotated up to point D, that the area of the right half of the circle can be transformed, and not before. Only then, will the right side of the circle be transformed into a

⁴ Something similar takes place inside of Raphael's *The Transfiguration*, where one scene must come into action before as the inverse of the other, while the whole is changing in the simultaneity of temporal eternity. See: [RAPHAELS TRANSFIGURATION HOW TO TRANSFORM THE TRAGIC INTO THE SUBLIME](#)

similar but inversed *sine-cycloid surface* when it becomes transformed under the right half of the cycloid.

Why are those two halves discontinuous with each other while their circular action is continuous? The only answer I can find at this time is because the whole of the cycloid can only be constructed by the two opposite halves of the circle in succession, one half at a time. The right side of the circle must rotate first and the left must rotate after, in chronological order.

OK, fine, but how can the same circular action do one thing and do something opposite immediately afterward in a continuous way? What sort of space-time is that? Can God go into two opposite directions without changing course? Why is there such a discontinuity within a continuous rotation? This may have to do with time reversal in temporal eternity.

Such a paradigm shift may be the reason why some people identified such a process of creativity as an “*odium theologicum*” (theological hatred).

ODIUM THEOLOGICUM
by Sam Walter Foss (1858-1911):

1*They met and they talked where the crossroads meet,
Four men from the four winds come,
And they talked of the horse, for they loved the theme,
And never a man was dumb.
The man from the North loved the strength of the horse,
And the man from the East his pace,
And the man from the South loved the speed of the horse,
And the man from the West his grace.*

*So these four men from the four winds come,
Each paused a space in his course
And smiled in the face of his fellow man
And lovingly talked of the horse.
Then each man parted and went his way*

*As their different courses ran;
And each man journeyed with peace in his heart
And loving his fellow man.*

*2They met the next year where the crossroads meet,
Four men from the four winds come;
And it chanced as they met that they talked of God,
And never a man was dumb.
One imagined God in the shape of a man.
A spirit did one insist.
One said that nature itself was God.
One said that he didn't exist.*

*They lashed each other with tongues that stung,
That smote as with a rod;
Each glared in the face of his fellow man,
And wrathfully talked of God.
Then each man parted and went his way,
As their different courses ran;
And each man journeyed with wrath in his heart,
And hating his fellow man.⁵*

Lyndon LaRouche showed that there are no fundamental conflicts between human beings, because there are no natural axiomatic discontinuities among human beings nor among nations. Nations go to war against each other because the Hobbesians of the world believe they are superior to other people. That axiomatic view is unacceptable. Here is what LaRouche proposed instead:

“First of all, what this represents is the legacy of two of the worst clowns in English-speaking history, Hobbes and Locke. Thomas Hobbes and John Locke. The idea that there has to be, that you have to run society, on

⁵ Sam Walter Foss, [ODIUM THEOLOGICUM](#).

the basis of some sort of inevitable, natural conflict among persons, nations, and peoples. Aren't we all human? I mean, even Henry Kissinger may qualify as human, under biological examination. Aren't we all human? Don't we all have a common interest in humanity? Don't we all have the same flesh and blood, and the same impulses and desires, really, fundamentally, as needs? Why should we be in conflict?

“Yes, we may have conflicts, but that doesn't mean this is a natural condition of man. This is the friction of trying to avoid conflict, as the Treaty of Westphalia of 1648 exemplifies that. And we would think, that after all that work that was done, including by Cardinal Mazarin, to bring about the Treaty of Westphalia, and you read the agreement itself, what it means: You would say, ‘This proves, and it proved to many in Europe until recently, that no matter how intense the war, how intense the struggle, there is always a way to find peace, and resolution, if you're willing to admit, that nations should love one another.’ Which is the Treaty of Westphalia: Nations should naturally tend to love one another.

“There is no such thing as a natural, axiomatic human conflict. There are human conflicts, but they are by their nature curable, because there's always a higher principle, lurking in the background. We are all human. None of us resemble apes. We're not. No ape can understand Gauss's fundamental theorem of algebra. And even though some people try to monkey around with it, that doesn't do it.”⁶

Such an accession to a state of temporal eternity is important to understand, because it is embodied in the very principle of the American Constitution, which calls for the “pursuit of happiness” of all of mankind not just Americans; that is, for the benefit of individuals in society who participate, in one form or another, in making humanity as a whole, immortal, by offering their own contributions and discoveries to the future of our species. Here is how Lyndon LaRouche stated the matter:

“So, this idea of creativity, as a distinction between man and the beast, is the meaning of ‘happiness.’ That is, we're all going to die, so therefore,

⁶ Lyndon LaRouche, [*Conflict Is Not the Natural Condition Among Men and Nations*](#), EIR, Vol. 50, No. 26, June 30, 2023, p. 26.

how can we take pleasure out of the fact that we're going to die? We can only take pleasure out of the fact we're going to die, when what we're doing while we're alive, somehow has permanent value for society: that we will live in the future, in that way, and practically, as our ancestors, our predecessors, who made discoveries of principle we share, live in us."⁷

CONCLUSION

The crisis of today is unique and timely, because it belongs to the temporal eternity of such axiomatic "gaps." This should not come as a surprise to anyone who has gone through and internalized such moments of crisis under the leadership of Lyndon LaRouche. As LaRouche himself reported on this matter:

"My specific, so far unique contribution to the science of physical economy, has been to demonstrate that the relevant types of axioms to be considered, are not only what would be more readily accepted as physical axioms, but also those axioms which are associated with validatable universal principles of Classical artistic composition. It is these axioms, and the technologies subsumed by their interrelationships, which must be recognized as "filling the gaps" between the shadows on the wall of Plato's Cave. Conversely, this view of such anti-entropic process-relations, defines the only meaning of axioms which is not, like today's generally accepted practice of financial accounting, the fruit of mere superstition.

"It is the higher battles, those won within the simultaneity of eternity, which are the form in which a deserved victory can be shared among all the worthy souls of humanity past, present, and future. Such, for the heirs of the gifts shared among a now globally extended form of modern European civilization, is the loving legacy we share with the Jesus Christ of Gethsemane. Let it be, in some meaningful way, as if they had not died."⁸

FIN

⁷ Lyndon LaRouche, [*The General Welfare Is The Constitution's Core*](#), EIR, Vol. 33, No. 6, February 10, 2006, p. 29.

⁸ Lyndon LaRouche, [*Lesson of the Cole Incident: Stop Privatizing Our Generals*](#), EIR, Vo. 27, No. 44, November 10, 2000, p. 53.