# THE ACROPOLIS OF ATHENS: THE CLASSICAL IDEA OF BEAUTY, PART II 

A geometrical construction for the equal-tempered musical system by Pierre Beaudry 11/7/16

## INTRODUCTION

"This geometrical performative and constructive method will take you to a higher type of understanding where you will discover a pathway by means of which you can never get lost."

Dehors Debonneheure
Following the uploading of THE ACROPOLIS OF ATHENS: THE CLASSICAL IDEA OF BEAUTY on my Galactic Parking Lot, last October 15, 2016, I had the occasion of discussing some of the axiomatics of the Parthenon Golden Section with one of our members who had been looking at other geometers and artists on the same subject, but who had come up with completely different results.

This gave me the opportunity to recall the nature of the axiomatics that I had been engaged in, 28 years ago, when I wrote the article on the Acropolis for The New Federalist of June $24^{\text {th }}$ and $30^{\text {th }} 1988$. Back then, I had demonstrated how to apply, geometrically, the Gauss-Riemann elliptic function of the Golden Section to the design of the Acropolis of Athens. However, I was not able, at that time, to discuss the axiomatics of the matter, neither to give the reader the means of making the crucial difference between the truth of a constructive proof and the fallacy of a curve-fitting demonstration. This difference is essential to master simply because it provides you with a pathway for discovering how to successfully construct an axiomatic transformation between two incommensurable manifolds.

This new report provides the reader with an opportunity to construct the equal-tempered-musical system with the constructive method of Jacob Steiner; that is, with a straight edge alone. You will know this is the right method when you realize that the pathway you took brings uniquely into coincidence what you have discovered with the way in which you have discovered it.

## 1. CONSTRUCTIVE GEOMETRY IS NOT CURVE-FITTING

At the beginning of the twentieth century, several authors, most notably, Samuel Colman, Clarence A. Coan, and Jay Hambridge had written books on the geometry of the Greek Parthenon in Athens which they related to living processes. You can find these two books on line by clicking on the following links for Samuel Colman and Clarence A. Coan, NATURE'S HARMONIC UNITY, G. P. Putman's Sons, 1912; and for Jay Hambridge, THE ELEMENTS OF DYNAMIC SYMMETRY, Dover Publications, New York, 1919.

What these three authors tried to solve was a crucial problem of constructive geometry that required a Riemannian solution to a problem that Vladimir Vernadsky had put before the world a few years later, in 1926, in his treatise on The Biosphere. One of the axiomatic questions that Vernadsky had raised, then, was how the human mind was required to relate to the non-living domain and the living domain as two fundamentally different manifolds projected together from within the higher manifold of the noosphere.

Those authors were not able to grasp the significance of that Vernadskian question because, instead of looking at the Riemann solution to this new approach, all three used Euclid's Elements as their choice of geometry for their approach to the problem. Their flaw came from a lack of concern for investigating some higher principle of mind that could subsume the domains of the living and the non-living and which would have permitted them to make the axiomatic jump from the higher domain of the living to the lower domain of the non-living. That is the singular
challenge which underlies the idea of discovering the creative design behind the Acropolis of Athens.

In all three cases, what was missing was the appropriate geometrical projection required to accomplish this purpose from the top down. Instead, their concern turned toward establishing connections, one-on-one, between the visual domain of sense perception and some mathematical proportion they wished to apply, particularly to the floor plan of the Parthenon of Athens, as if that would secure the truth of the matter. The approach they chose for their purpose can be characterized as "curve-fitting." The question the reader may be asking is: "What is wrong with this sort of curve-fitting approach?"


Figure 1 The Hambridge model for the floor plan of the Parthenon.
Let's take the example of two different models for the floor plan of the Parthenon and look at the right and wrong ways to design a plan for it, from the vantage point of a higher geometry. I will first examine the flat construction plan of the Parthenon by Hambridge, and then, the curve-fitting projection by Colman
and Coan. Then, I will examine the Platonic circular action plan that a contemporary designer, Athanasios Angelopoulos, has published recently.

On the one hand, there is nothing wrong with the Hambridge construction, as such. See Figure 1. Hambridge applied the traditional Greek method of using compass and straight edge alone and constructed his floor plan based on the rootfive rectangle by rotating the radius of a circle centered on the half point of a side of the square and touching the opposite angle of the same square. Hambridge wrote: "This is the root-five rectangle and its end to side relationship is as one to the square root of five, 1: 2.236 ; the number 2.236 being the square root of five. Multiplied by itself this number equals 5."(Jay Hambridge, THE ELEMENTS OF DYNAMIC SYMMETRY, Dover Publications, New York, 1919, p. 26.) The disadvantage of this constructive method is that it does not identify where the projection is coming from and it implies that the proof is in the numbers. That is another fallacy of composition.

The question is: How can this perfectly correct Pythagorean circular action construction of the square root of five be generated from a higher principle which would account for the idea of growth of living and non-living processes alike? The answer to that question lies in the conical-spiral principle of least action which is the generative source of simple circular action in the plane. Keep this idea in the back of your mind while we go through the following example.

Next, take the Colman and Coan model of a projection of the floor plan of the Parthenon against two inverted pentagons inscribed in a circle. (See Figure 2) What is wrong with that sort of curve-fitting process?

It is unfortunate that Colman did not know about the original Greek construction of compass and straight edge or about the Gauss-Riemann higher geometrical approach to constructive geometry.

As Colman wrote in the very first paragraph of his book: "Proportion is a principle in nature which is a purely mathematical one and to be rightly interpreted by man through the means of geometry; therefore, geometry is not only the gate way to science, but it is also a noble portal opening wide into the realms of art."
(Samuel Colman and Clarence A. Coan, Nature's Harmonic Unity, p. 1.) That's where he made his mistake, the same mistake that Hambridge had made. Geometry may be the gate to science and art respectively, but proportion is not purely a mathematical principle. That is incorrect.


Figure 2 Curve fitting of the ground plan of the Parthenon with pentagons by Samuel Colman and Clarence A. Coan, Nature's Harmonic Unity.

Proportionality is an offspring of the creative process of mind. What Colman and Coan were trying to do may be noble, but the sovereign is not mathematics. Such a mathematical illusion does not pass the test of time and is definitely not
demonstrable constructively or by curve-fitting. The main problem is that these authors missed the higher performative dimensionality of constructive geometry. They limited themselves to the flat universe of Euclid and excluded the higher domain of a mental projection. Why is this wrong?

The Figure 2 projection of inverted pentagons inscribed in a circle tells you nothing about the actual Parthenon construction, except that the reader is made to focus his attention onto a one-on-one connection between the floor plan and a purely formal and abstract idea. There are no constructible connections between them. Why, then, did Colman and Coan do something so unnatural?


Figure 3 The Doric circle inscribed in the square represents the Platonic model of construction for the floor plan of the Parthenon stylobate in the proportion of $9: 4$. (Athanasios G. Angelopoulos, Metron Ariston, Athens, 2003.)

Lastly, let's take a third construction for the floor plan of the Parthenon and consider the difference with the two previous examples. See Figure 3. Ask yourself: why did the ancient Greeks establish their geometric constructions on the compass and the straight edge alone? Why did they start with "plane" geometry? Compare the Angelopoulos construction with the previous two illustrations of Figure 1 and Figure 2. What is included in this figure and is absent from the other two?

This Angelopoulos idea forces you to ask yourself a question that the other two did not ask and that the Ancient Greek method excluded from "plane" geometry. And yet, the question is the one that its builders, Iktinos and Mnesikles, asked themselves, when they erected that Temple to Athena during the Fifth Century BC: How can you express a geometrical measure of growth which is capable of reproducing itself within its own construction? In other words, what is the geometrical principle that would not only be proper for the whole of the Acropolis, but would be reflected in all of the temples erected on it, as well as in every part of each of those temples? The geometrical answer is a very nice and uncomplicated process of discovering the principle behind the circular action intersection between the sides and the diagonals of a square. What are the implications of such a discovery? The implications are that the process of such an interaction implies the creative process of the human mind.

This is the method that Plato used in The Meno dialogue for the purpose of making the slave boy discover how to double the area of the square; that is, how to make something grow by reflecting everywhere the process of its development in a self-similar manner. In other words, the construction has to include something "mental" which reflects the creative process of growth as opposed to some mathematical calculation. Therefore, mathematics must be replaced by constructive performative action. See my report: WHAT SHOULD HAVE BEEN THE FUTURE

The simplest form of this "mental" circular action construction was the discovery of the Pythagorean Theorem as a means of making an area grow through the diagonal of its original figure. That was very likely how the floor plan, the front
elevation, and the metope-triglyph relationships were originally designed for the Parthenon.

So, Figure 3 shows how you would go about realizing that intention by simple circular action construction, as the ancient Greeks used to do, with a compass and straightedge alone. Thus, the four intersection points $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ generated by the double circular action of the diagonals of the four inscribed squares and by the sides of the larger square $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$, locate beautifully the floor plan of the Parthenon, without curve fitting and without the root of five rectangle. The floor plan is discovered from the proverbial outside, as a variation of Meno's discovery of doubling the square. Similarly, any new discovery can and should only be made from the proverbial outside of the box and from the top down; that is to say, from conditions established by external boundary conditions. Thus, the method implies the necessity of a higher manifold of projection; a doublyconnected manifold.

## 2. THE AXIOMATIC JUMP FROM THE CONICAL-SPIRAL PROJECTION: A NEW PATHWAY OF UNDERSTANDING

During the early 1980's, I spent an extended period of four years working out an hour of geometry every morning, before my daily deployments, in order to master the constructive method of Jacob Steiner that I developed for my Acropolis report: THE ACROPOLIS OF ATHENS: THE CLASSICAL IDEA OF BEAUTY

As a result of that work, I was able to construct the geometric spiral with corresponding conical projections for the equal-tempered musical system, and the Gauss elliptical functions for the arithmetic-geometric mean idea in music. This work not only provided me with a coherent gestalt for generating harmonically ordered musical intervals, but also a completely new and better way of understanding how the physical universe works in the small as in the large.

Once the work was done, Mark Fairchild wrote up for me a computer program in order to generate the graphic illustrations for my 1988 New Federalist article. The computer programs are now lost, but, the most important aspects of the footprints of that discovery are included in Figures 8a and 8b of my report. See Figures $4 a$ and $\mathbf{4 b}$ of this report below.


Figure 4a The constructive method of the equal-tempered system. A correction has to be made in these 1988 illustrations of the ordering process of the scale. The spiral action is rotating counterclockwise from apex to the base of the cone, and not the other way around as it is illustrated here. In other words, the process of change goes from F to G and not from G to F , which means that the position of the notes should have been inversed and $F$ should have been in the position of $G$.

Therefore, the dynamic dissymmetry one should be looking for, between the Parthenon and living processes, is not going to be found by curve-fitting in the plane; it is going to happen when you start asking yourself why curve-fitting doesn't work and where the mapping process of a plane projection comes from. The question is: How can you find the pathway that leads to discovering a higher type of projective geometry which is not based on curve-fitting? The irony of the answer is that the discovery of that higher geometry is nothing but the pathway that leads you to its discovery.

Once you discover this, then you know, without the shadow of a doubt, that this is the way to go anywhere and come back from without ever getting lost. That's why I have always recommended this method of discovery, not only as a way of looking for something but also, most importantly, as a way to discover the pathway that leads you to it and back from it. If you discipline your mind to do that, you will begin to discover a higher geometry as soon as you start looking for it.

The fundamental question, here, is how to relate two manifolds of incommensurable proportionality; that is, the conical-spiral action manifold and the simple circular action manifold. The answer to that question provides us with a basis for understanding how the nineteenth century mind worked and how the mind of the early twentieth century failed to discover the required epistemology connecting the economics of nature and mind that Lyn has been the only one to discover and develop, during the 70's and 80's, in the footsteps of Riemann and Gauss.

My construction of Figure 4a and Figure 4b intend to give the reader the beginnings of an answer to that epistemological question by providing elements of a pedagogical exercise reflecting a complex function which also applies to itself, as it develops itself; that is to say, by providing you with a performative instrument which you can reconstruct by inversion in order to experience the discovery of how an axiomatic change works in your own mind, from the top down.


The Parthenon $i s$ besed on a very specific spiral progression which cant be expressed in 1 iving processes by the Fibonacei seriess. This is the reason why the triglyph-metope relationship above the architrave defines the intercolumnization in the same golden rectangle proportion as the front elevation is to the entire floor plan of the Farthemon.

Figure 4b The higher domain of conical spiral action determining the front elevation of the Parthenon in Golden Proportion and in accordance with the equal temperament of the Bach musical system and the Lydian ordering of G, F, and C. The dominant, subdominant, and tonic, thus, reflect how the higher musical dimensionality of the golden section is being generated as a complex conic section
which projects those three keys together in the form of architectural beauty. How can that be?

In other words, you are not looking at an object being projected before your eyes; you are looking at how your mind is projecting the ordering of the principle of classical artistic composition and how that takes place by least action. That's the change which has to take place in your mind; that is, an axiomatic transformation which can only be discovered by constructing it, and proving it to yourself.

What you are looking for, therefore, is not a formal proportionality, but an incommensurable connection between your mind and the mind of another to a lawful ordering process which applies to architecture through music by means of the Lydian divisions of the octave of the musical scale. Thus, you are looking at how incommensurable dissonant connections get resolved by discovering the least action process that fuses and connects minds together. Applying this principle generates a significant increase in energy-flux-density, which is necessary for the growth of human knowledge and of the living conditions of mankind in general (i. e. LaRouche economics). That is how the musical ordering principle of J. S. Bach became, for me, the incommensurable glue that cemented epistemologically both science and artistic composition, back in 1988; that is, in accordance with what Lyn was developing, then, with his Riemannian economic principle.

Such is the Lydian ordering that takes hold of your mind when you project an incommensurable mapping between two or more domains of knowledge taken together. Just as in the process of changing key signatures inside of a musical composition, the musical dissonances, or more specifically the Lydian dissonances of the Bach process of the Bel Canto voice register shift, become the congruent living measures of harmonic, arithmetic, and geometric proportionality, which generate the appropriate axiomatic transformations from one domain to another domain, as in the particular case of the higher manifold of least-spiral-action projection onto the lower plane of the visible manifold. This is what I discovered in 1988, with the help of Lyn, and what I have applied to this conical-spiral construction in Figure 8a and 8b of my Acropolis article.


Figure 5 Model of the Parthenon by Pierre Beaudry.

If you want to look at a simple footprint of it, just take the ratio of 9:4 ratio of the floor plan of the Parthenon and consider it as the shadow of the proportionality of J.F. Bach's Lydian divisions of the octave. You should be able to see, as if reflected from the dimly lit wall of Plato's Cave, the shadow of an ordering of the three sets of minor thirds progressing over four least action steps in the equal-tempered system; that is, the ordering principle behind the relationship which links the dominant $G$ and the subdominant $F$ to the tonic $C$. The best way to hear this is to play the three appropriate Lydian sets of minor thirds generating successively the keys of G, F, and C, descending on the keyboard. That's how you generate the musical golden section. That is what Bach has established as the generating principle of the First Prelude in C Major of the Well-tempered System.


Figure 6 Peplos Ceremony. Center piece of the east frieze of the Parthenon, ca. 447-433 BC.

## 3. THE MAIN STEPS FOR THE GEOMETRICAL CONSTRUCTION OF THE EQUAL-TEMPERED MUSICAL SYSTEM

(You are allowed: a T-square, a right-angle triangle, a compass and divider, a pencil, and an eraser. No mathematical measuring or calculation is required.)


Figure 7 Construct the skeleton of a right angle upside-down cone of about the size of your extended hand, and make a perpendicular base of the same length as the height. Divide the whole cone in half, and the bottom half of the cone, in half again. Identify those two halves by circular lines marking the boundaries of two musical octaves: C-C' (128-256) and C'-C" (256512). Then, draw two elliptical lines across the two octaves. The two elliptical lines have created two new points at the intersection of the cone's axis. Those two points mark the location of the harmonic means of those two C octaves, respectively, and locate the subdominant octave of F. This first step tells you that the whole process will follow the circle of fourths. Thus, you can already identify in your mind the order in which you will discover all of the notes of the two octaves of C, by constructing the counterclockwise spiral locations of C, F, Bb, Eb, $\mathrm{Ab}, \mathrm{Db}, \mathrm{F} \#, \mathrm{~B}, \mathrm{E}, \mathrm{A}, \mathrm{D}, \mathrm{G}$, in that order. The process follows the circle of fourths because the ratio of the circle of fourth and the ratio of the harmonic mean of the spiral action moving counterclockwise around the cone are the same: 5:7 ratio.


Figure 8 Draw two circular lines at the two new points across the axis of the cone and identify them as the octave of F and F '. Remember that the fifth equaltempered half-tone upscale after F , is Bb . Draw a circular line Bb perpendicular to the axis of the cone at the harmonic mean location of F. After the construction of those first few steps, you have to pause, because you won't know where you are going to go next. You have to look for a new octave to build, but how, which one, and where? You are stuck. Try to construct that new octave in your own mind before you turn the page to Figure 9. However, note that you already know, by construction, the location of the harmonic mean, and that the harmonic mean of
each note always intersects one of the two ellipses of the two octaves of C-128 and C'-256.


Figure 9 Next, you have to create a new imaginary projection upwards from the apex of the cone to the point of intersection of circular line Bb with the elliptical line of the lower octave of C-128-C’-256. Project that imaginary line through that point to $\mathrm{Bb}^{\prime}$ which is intersecting the elliptical line of the upper octave of $\mathrm{C}^{\prime}-256$ $\mathrm{C}^{\prime \prime}-512$. Identify that point as Bb ' and draw a parallel circular line at that point. You have now located the upper range of the $\mathrm{Bb}-\mathrm{Bb}$ ' octave. Draw an elliptical line across the cone joining the circles of Bb to Bb '. You have now created a new harmonic point Eb' through the axis of the cone. Draw a circular line at point Eb' across the axis of the cone. Project another imaginary line from the intersection point Eb' with the octave of C'-C" down to the apex of the cone. The new point on
the lower elliptical octave of C-C' will mark the range of the octave of $\mathrm{Eb}-\mathrm{Eb}$ '. Continue this process until you have found and marked all of the ellipses and circles crossing the two original octaves $\mathrm{C}-\mathrm{C}^{\prime}$ and $\mathrm{C}^{\prime}-\mathrm{C}^{\prime \prime}$. The imaginary apexprojection through the entire cone makes you discover all of the half tones of the equal-tempered system with a straight edge alone.


Figure 10 Construction of all of the circular lines and elliptical lines representing the intervals of two octaves from C-128 to C"-512 of the equal-tempered system. Locate all of the notes of the two octaves at the appropriate intersections of circles
and ellipses in the higher manifold before projecting them down, orthographically, onto the lower manifold of the circular plane.



Figure 11 Although they are not necessary for this construction, you are allowed three concentric circles in proportion of $2: 1$ in the plane. Construct the lower-plane manifold in accordance with the circular and elliptical lines of Figure 10 and you will discover that the projection of this conical-spiral-action is entirely based on the power of two, the fundamental power of growth in the universe. Every note must be dropped orthographically from the top down as shown for B' and G'. See my report: A GEOMETRICAL METHOD TO INVESTIGATE THE FUTURE.


Figure 12 The identification and location of the 24 notes of the two octave on the logarithmic spiral of the cone requires that the connection be made first between the two manifolds, the higher (cone) and the lower (circle) manifolds. All such 24 notes of the two octaves must be projected from the higher manifold through orthographic lines onto the lower manifold. Consequently, the twelve radial divisions of the lower manifold and their orthographic connections with the higher manifold must first be drawn in the plane, and afterward projected back into the cone. Lastly, you can draw the logarithmic spiral as a result of connecting all of the
notes together. You are not required to construct the arithmetic spiral, because it is not required for this equal-tempering exercise.


Figure 13 Draw three concentric circles in proportion of $2: 1$ and project the notes orthographically from the surface of the cone onto the 12 radii of the circular plane. Lastly, draw the logarithmic spiral in the plane as a result of connecting all of the notes together. Thus, within the range of two octaves in proportion of 2:1, you have constructed the equal-tempered musical with a straight edge alone.


Figure 14 How the Planets of our Solar System correspond approximately to the intervals of the Equal-Tempered System. Here, the octave of C-256-512 is made to correspond to the astronomical units of Mercury and Neptune, respectively. (See Figure 15) The divisions of Mercury, Earth, Jupiter and Neptune, represent the values for doubling the cube.

THE PLANETARY ORBITS AND THE EQUAL-TEMPERED MUSICAL SYSTEM
by WILLIAM BOHDAN

| PLANETS | ASTRO. UNITS | $\begin{aligned} & \text { Log. } \\ & \text { 10X } \end{aligned}$ | ADDED CONSTANT | MULTIPLE CONSTANT | $\begin{aligned} & \text { CYCLE } \\ & \text { EQUIVALENT } \end{aligned}$ | MUSICAL CYCLES | PLANETS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MERCURY | (P) 0.310 | 0.5086 | +2.496 | x 128.8 | 255.97 | $\mathrm{C}=256$ | MERCURY |
| MERCURY | (A) 0.470 | 0.3279 | " " | " | 279.25 | C\#=271.22 | MERCURY |
| VENUS | (P) 0.715 | 0.1457 | " | " " | 302.72 | $\mathrm{D}=287.35$ | VENUS |
| VENUS | (A) 0.725 | 0.1397 | " | " | 303.49 | $\mathrm{Eb}=304.44$ | VENUS |
| EARTH | (P) 0.983 | 0.0074 | " | " | 320.52 |  | EARTH |
| EARTH | (A) 1.017 | 0.0073 | " " | " " | 322.42 | $\mathrm{E}=322.54$ | EARTH |
| MARS | (P) 1.379 | 0.1396 | " " | " " | 339.46 | $\mathrm{F}=341.72$ | MARS |
| MARS | (A) 1.661 | 0.2204 | " | " " | 349.86 |  | MARS |
| ASTEROIDS | (P) 2.2 | 0.3424 | " | " " | 363.32 | F\#=362.04 | ASTEROIDS |
| ASTEROIDS | (A) 3.6 | 0.5563 | " " | " " | 393.13 | $\mathrm{G}=383.57$ | ASTEROIDS |
| JUPITER | (P) 4.95 | 0.6946 | " " | " " | 410.95 | $\mathrm{Ab}=406.37$ | JUPITER |
| JUPITER | (A) 5.45 | 0.7364 | " " | " " | 416.33 |  | JUPITER |
| SATURN | (P) 9.006 | 0.9545 | " | " " | 444.43 | $\mathrm{A}=430.54$ | SATURN |
| SATURN | (A)10.074 | 1.0032 | " " | " " | 450.69 | $\mathrm{Bb}=456.14$ | SATURN |
| URANUS | (P) 18.288 | 1.2622 | " | " | 484.05 | $B=483.26$ | URANUS |
| URANUS | (A) 20.092 | 1.3030 | " | " " | 489.31 |  | URANUS |
| NEPTUNE | (P) 29.799 | 1.4742 | " " | " " | 511.36 |  | NEPTUNE |
| NEPTUNE | (A) 30.341 | 1.4820 | " | " " | 512.37 | $\mathrm{C}=512$ | NEPTUNE |
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| PLANETS | ASTRO. UNITS | $\begin{aligned} & \text { Log. } \\ & 10 \mathrm{X} \end{aligned}$ | ADDED CONSTANT | MULTIPLE CONSTANT | CYCLE EQUIVALENT | MUSICAL CYCLES | PLANETS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MERCURY | (P) 0.310 | 0.5086 | +2.496 | x 128.8 | 255.97 | $\mathrm{C}=256$ | MERCURY |
| MERCURY | (A) 0.470 | 0.3279 | " | " | 279.25 | C\#=271.22 | MERCURY |
| VENUS | (P) 0.715 | 0.1457 | " | " " | 302.72 | $\mathrm{D}=287.35$ | VENUS |
| VENUS | (A) 0.725 | 0.1397 | " " | " " | 303.49 | $\mathrm{Eb}=304.44$ | VENUS |
| EARTH | (P) 0.983 | 0.0074 | " " | " " | 320.52 |  | EARTH |
| EARTH | (A) 1.017 | 0.0073 | " | " " | 322.42 | $\mathrm{E}=322.54$ | EARTH |
| MARS | (P) 1.379 | 0.1396 | " | " " | 339.46 | $\mathrm{F}=341.72$ | MARS |
| MARS | (A) 1.661 | 0.2204 | " | " " | 349.86 |  | MARS |
| ASTEROIDS | (P) 2.2 | 0.3424 | " | " " | 363.32 | $\mathrm{FH}=362.04$ | ASTEROIDS |
| ASTEROIDS | (A)3.6 | 0.5563 | " " | " " | 393.13 | $\mathrm{G}=383.57$ | ASTEROIDS |
| JUPITER | (P) 4.95 | 0.6946 | " " | " " | 410.95 | $\mathrm{Ab}=406.37$ | JUPITER |
| JUPITER | (A) 5.45 | 0.7364 | " " | " " | 416.33 |  | JUPITER |
| SATURN | (P) 9.006 | 0.9545 | " " | " " | 444.43 | $\mathrm{A}=430.54$ | SATURN |
| SATURN | (A)10.074 | 1.0032 | " " | " " | 450.69 | $\mathrm{Bb}=456.14$ | SATURN |
| URANUS | (P) 18.288 | 1.2622 | " " | " " | 484.05 | $\mathrm{B}=483.26$ | URANUS |
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| NEPTUNE | (A) 30.341 | 1.4820 | " " | " " | 512.37 | $\mathrm{C}=512$ | NEPTUNE |

Figure 15 Planetary System calculated by William Bohdan. The top illustration shows the divisions of the equal-tempered musical octave by the Arithmetic-Geometric-Harmonic Mean proportionality from F, F\#, G, and the bottom
illustration shows the divisions of the octave of C-256-512 and its Lydian Minor Thirds.


FIRST 4 MEASURES OF MOZART 'S FANTASY IN C- 475 FIGURE 25.


MEASURES 161-164 OF MOZART'S FANTASY SONATA - 475.

Figure 16 The Lydian Golden Section principle applied to the opening measures of Mozart's Sonata-475.

## CONCLUSION



Figure 17 Luca della Robbia, Cantoria, 1431/1438.

As the famous Florentine sculpture by Luca della Robbia demonstrates, by construction of a special sort of conical-spiral least action process, (See Figure 17), the observer is able to examine the axiomatic passage of the register shift of the Soprano voice change in precisely the harmonic-geometric-arithmetic range of Bel Canto singing of the middle-C octave. How do I know that, you ask?

The reader is actually capable of seeing with his own physical eyes what is going on in Rubbia's mind. What Luca della Rubbia knew to be true is that the axiomatic passing of the register change of the middle-C octave must be located in the passage from subdominant F to the dominant G of the middle-C octave at 256. This can be recognized in the faces of the three central boys.

Observe the three boys, from right to left, and you can see the characteristic changes that take place inside of their heads, when a boy's voice changes by going from the chest register to the head register. Although you cannot hear what is being
sung, your eye can identify the differences of registering in the voice simply by locating the muscle tension of their foreheads, noses and mouths.

The voice of the boy on the right is located in the lower chest register, the voice of the boy in the center is moving through the register shift range, somewhere between F and F , and the voice of the boy on the left is singing in the higher head register above $\mathrm{F} \#$ after middle-C. From the vantage point of epistemology, this vocal axiomatic change comes from the same conical function connecting the lower and the higher domains of the Solar System through the register shift located at the asteroid belt.

As Lyn demonstrated in A MANUAL ON THE RUDIMENTS OF TUNING AND REGISTRATION, the best experimental proof demonstrating the existence of an increase in energy-flux density is located in the experiment of your own mind. And, the most effective demonstration of this performative discovery of principle is to actually cause an axiomatic change by constructing it, yourself, with your own voice in a bel canto exercise. Try it with a competent voice teacher, and you will see.

The point to retain, here, however, is that such a form of complex spiral action has a deliberate upward directionality which increases the energy-flux density of the mind as an expression of universal progress. Although the experiment which takes place is not about something that was missing in the human singing voice before the axiomatic event of the register shift, it is rather something that existed as a potential that the voice had always been able to project before.

Think of the range of change as being defined musically by means of what Lyn located as the axiomatic turbulences of the soprano singing voice range between the sub-dominant and the dominant above the key of Middle-C $=256$. As Lyn put it:
"The value of the F , as the subdominant interval, and G as the dominant, for the key of (Middle-C $=256$ ), has a very precise musical significance, best understood from the standpoint of constructive geometry.

For the moment, it is sufficient to indicate, that a key which divides the octave at subdominant to dominant, in this way, is the most natural of keys from the standpoint of physics and principles of classical composition. The congruence of the division of the octave by singing-voice register, with the passage from the sub-dominant to dominant is crucial for understanding the interconnectedness of singing with definitions of well-tempered scale. This interconnectedness is the ground-principle which distinguishes human music from the abstract music of such dead objects as musical instruments. This is what defines human music, the only real music, as situated within a doublyconnected manifold." (Lyndon LaRouche, TRUTH IS BEAUTY AND BEAUTY IS TRUTH: UNDERSTANDING THE SCIENCE OF MUSIC, 9/9/1986, p. 38-39.)

The question this raises is: what is the underlying musical ordering which takes all of the soprano and tenor voices through such a change in the region of the subdominant F to the dominant G of the middle-C octave range? What does such an ordering suggests for the musical system as a whole? What is the significance of dividing the whole musical system by half at the middle-C octave? And, what is the meaning of the division of this middle-C octave, by half again? What is the result of this division? You are right. It's the Lydian range. The ordering which divides the middle-C octave by half and by half again goes from C to F \#, then from Eb and to A. Could we have centered our construction on another octave? No. Why not?

Lyn says clearly that the value for F as the sub-dominant and of G as dominant must be only for the middle-C at about 256 . The point he makes is absolutely true, this is the most natural octave for physics and for the great majority of sopranos and tenor voices, thus, for a great number of human beings on this planet. But, is there another reason to choose this octave of middle-C among all others? Yes. Ask yourself the following question. When is the best time to make a change in a continuous process? At the half-way mark. Why? Because the halfway mark is the last chance to change. Once you have passed that point, you are coming back to yourself. In other words, all living processes divide by half
once, and then, half of the half, one more time, without exception, because all life follows the law of change, and must change before it is too late.

In that sense, life itself is based on the Lydian form of division, and the reason for the Lydian divisions of the octave to establish middle-C as the key octave for the construction of the system as a whole, is because there is not such a thing as "absolute pitch." The only "absolute," here, is the division of the whole by half, and by half of the half, again. That is why this form of Lydian division worked so well in the conical-spiral projection that I have used. Day and night time is always also better divided by half and half the half. The half-way mark is always where the curvature of everything changes; the before and after, the front and the back, the right and the left, and so forth. That's the effect of the golden section of the conical function. Thus, the classical idea of beauty of this method does not lie in some romantic idea of looking good or sounding good. It is the result of a living process of proportional division which generates the golden section in the plane as an expression of its universal consistency of growth from a higher domain.

Thus, the subject matter of THE ACROPOLIS OF ATHENS: THE CLASSICAL IDEA OF BEAUTY was never really about the Parthenon of Athens, as such, but about how the creative mind works. The construction of the Parthenon, itself, was a pretext for attempting to discover what was the true intention behind the enigmatic Peplos ceremony, depicted as the central scene of the East frieze and representing the high point and finale of the procession around the temple in honor of the goddess Athena. The Peplos is where the builders of the Parthenon have hidden what could not be seen with your physical eyes; that is, a folded representation of the Platonic higher intelligible domain of the soul of the world enveloping the universe as the highest expression of a creative mind. See my report: THE CURVATURE OF THE PARTHENON, PART II.

Similarly, the purpose of this report was to discover the constructive proof of an epistemological pathway which is hidden in the axiomatic foundations of the human mind's ability to change and improve others, harmonically. That intention has been fulfilled. Now, isn't that what the classical idea of beauty is all about?

## FIN

